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ON THE BROADENING OF SPECTRAL LINES.

By A. A. MICHELSON.

To ACCOUNT for the finite width of the spectral lines of a substance emitting approximately homogeneous radiations, the following hypotheses have been proposed:

1. As a consequence of Kirchhoff's law "the ratio of brightness of two immediately contiguous portions of a discontinuous, bright-line spectrum constantly decreases, if the number of luminous strata is multiplied or if the coefficient of absorption of the single stratum is increased, until the value is reached which, for the same wave-length and the same temperature, corresponds to the ratio in the continuous spectrum of a body completely opaque for the given thickness." [1]

2. The direct modification of the period of the vibrating atoms in consequence of presence of neighboring molecules.

3. The exponential diminution in amplitude of the vibrations due to communication of energy to the surrounding medium, or to other causes.

4. The change in wave-length due to the Doppler effect of the component of the velocity of the vibrating atom in the line of sight.

[1] SCHEINER, *Ast. Spect.*

To these the following causes may be added:

5. The limitation of the number of regular vibrations by more or less abrupt changes of phase amplitude or plane of vibration, caused by collisions.

6. The possible variations in the properties of the atoms within such narrow limits as to escape detection by other than spectroscopic observations.

This last hypothesis is added chiefly for the sake of completeness, but it seems highly improbable that this cause could be of great importance, and serious objections at once occur, if we attempt by its means to explain the effects of temperature and pressure.

The most promising method of assigning to these different causes their proper measure of importance is to investigate such effects of temperature and pressure from the theoretical as well as the experimental side, and until this is accomplished only vague and unsatisfactory conclusions may be expected. The great importance of such a study, and the interest in it which is shown by such able contributions as those of Wiedeman, Ebert, Rayleigh, and recently, of Jaumann and Galitzin justify the prediction that a complete and satisfactory theory will be forthcoming in the near future.

Meanwhile it may be of interest to consider some results which have been reached in the present paper as a result of several months' labor. That these are in some cases very imperfect is freely admitted, but it is hoped that further experiment may give more definite information—and at least that this contribution will present the essential points in such a manner as to open the work to discussion and criticism.

The objections to the hypotheses numbered 1 and 3 have been considered by Galitzin, Kayser, and others, and these are, I believe, fatal unless the main contention that they are the essential causes be abandoned. Very probably they produce secondary effects which may be traced when the complete theory is developed.

In regard to 4, I cannot agree with the conclusions of Galitzin.

On the contrary the evidence of a very large number of experiments points unmistakably to effects of motion of the molecule in the line of sight. Lord Rayleigh[1] has shown from the standpoint of the kinetic theory that the motion of the molecule in the line of sight can produce a broadening of the lines, and it seems probable that this is the chief if not the only effective cause in operation when the density of the radiating body is low.

Thus in the case of hydrogen it has been shown that the increase in width of the red line above its width at zero pressure is proportional to the pressure,[2] but that even at zero pressure (pressures below $0^{mm}.5$) it has a definite width which is of the order of magnitude required by Rayleigh's investigation on the assumption of a molecular velocity of from 2000 to 4000 meters per second. It has also been pointed out that this limiting width of the spectral lines on this assumption should be proportional to the square root of the absolute temperature and inversely proportional to the square root of the molecular weight of the radiating substance.

The following table gives a list of substances of molecular weights varying from 1 to 200 v_1 and v_2 denote molecular velocities, the former deduced from the inverse ratios of the square root of the molecular weight; the latter calculated from the visibility curves. The agreement with theory while far from perfect is still—considering the difficulties of the experiments, and especially the uncertainty in the temperatures—too striking to be accidental.

It may therefore be conceded that while the causes 1, 3, 4 and 6 together or separately may and probably do produce an appreciable effect, they are generally insufficient to account for the broadening of the spectral lines.

[1] *Phil. Mag.*, April, 1889.

[2] *Phil. Mag.*, September, 1892.

Substance	At. Wt.	v_1	v_2
Hydrogen	1	2000	1500
Lithium	7	800	1200
Oxygen	16	500	800
Sodium	23	400	400
Magnesium	24	400	650
Iron	56	260	500
Cobalt	59	260	560
Nickel	59	260	500
Copper	63	250	450
Zinc	65	250	450
Palladium	106	190	250
Silver	108	190	250
Cadmium	112	190	220
Gold	196	140	225
Mercury	200	140	140
Thallium	204	140	110
Bismuth	210	140	150

The following is an attempt to deduce the value of the broadening of the spectral lines (or at least to find its order of magnitude) on the hypothesis 5.

In Fourier's formula,

$$\phi(x) = \frac{1}{\pi}\int_0^\infty du\ (C \cos ux + S \sin ux),$$

in which

$$C = \int_{-\infty}^{\infty} \phi(v) \cos uv\, dv \text{ and } S = \int_{-\infty}^{+\infty} \phi(v) \sin uv\, dv,$$

the second member represents a group of perfectly homogeneous trains of waves.

If therefore $\phi\ (x)$ represents the train of waves incident on a prism, the intensity of its spectrum will be

$$\psi^2\ (u) = C^2 + S^2.$$

Suppose the incident light to consist of a limited number of exactly equal waves corresponding to the number of vibrations emitted by the molecule between two collisions.

If ρ = free path of the molecule,
v = velocity of translation,
V = velocity of light,
r = length of a single train of waves,

then, between the limits $-\frac{1}{2}r$ and $\frac{1}{2}r$ we have

$$\phi(x) = a \cos mx + b \sin mx$$

$$C = a\int_{-\frac{r}{2}}^{+\frac{r}{2}} \cos mv \cos uv\, dv = a\,\frac{\sin \frac{1}{2}(m-u)r}{m-u} + a\,\frac{\sin \frac{1}{2}(m+u)r}{m+u}$$

$$S = b\int_{-\frac{r}{2}}^{+\frac{r}{2}} \sin mv \sin uv\, dv = b\,\frac{\sin \frac{1}{2}(m-u)r}{m-u} - b\,\frac{\sin \frac{1}{2}(m+u)r}{m+u}$$

These terms are all very small (since m is large) except for values of u which differ but little from m or from $-m$. Leaving negative values of u out of consideration, and replacing $u-m$ by $2\pi n$ we have

$$\psi^2(n) = \frac{(a^2+b^2)}{4}\,\frac{\sin^2 \pi n r}{\pi^2 n^2}$$

This represents the distribution of intensities in the spectrum of such a limited train of waves, and if the free path and the velocities of all the molecules were the same, the "width" of the resulting spectral line might be obtained from this expression.

Fig. 1 represents the intensity curve, and the "width" of the line may be taken equal to

$$\pi n r = \pi, \text{ whence}$$

$$n = \frac{1}{r} = \frac{1}{\rho}\frac{v}{V}, \text{ or in wave-lengths,}$$

$$\delta_2 = n\lambda^2 = \frac{\lambda^2}{\rho}\frac{v}{V}.$$

In the case of hydrogen at a pressure of 100 mm. we may take $\lambda = 6560 \times 10^{-10}$ $v = 3000$ $V = 3 \times 10^8$ $\rho = 7500 \times 10^{-10}$, whence $\delta_2 = .057$ tenth-meter.

The corresponding quantity obtained by experiment (p. 295, *Phil. Mag.*, September 1892) is $2\,(0.128 - 0.048) = 0.16$ tenth meters which is of the same order of magnitude as the theoretical value.

As experiment has shown that in every case thus far examined the width of the spectral lines diminish with the pressure in an approximately linear proportion towards a constant

limiting value, we may for a first approximation derive a formula for the actual width of the line on the assumption that this is the sum of the separate widths due (1) to the motion in the line of sight (2) to the limitation of the free path. The latter has just been given. The former is $\frac{2}{\pi} \times \frac{l_2}{\Delta} \lambda^2$ if Δ is the distance at which the "visibility curve" [1] falls to half its maximum value.

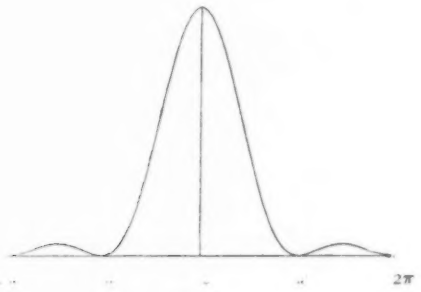

FIG. 1.

Rayleigh's formula as modified by the definition of visibility given in a former paper [2] becomes

$$V = e^{-\pi \left(\frac{\pi X}{\lambda} \frac{v}{V}\right)^2}$$

$$\text{which gives } \Delta = \frac{1}{\pi} \sqrt{\frac{l_2}{\pi}} \frac{V}{v} \lambda, \text{ whence}$$

$$\delta_1 = 2 \sqrt{\pi l_2} \frac{v}{V} \lambda = 3 \frac{v}{V} \lambda, \text{ nearly.}$$

$$\text{We have then } \delta = \delta_1 + \delta_2 = \frac{v}{V} \lambda \left(3 + \frac{\lambda}{\rho}\right).$$

If v = molecular velocity at absolute temperature θ and v_0 the corresponding velocity at θ_0, m the molecular weight; d, the actual density and d_0 the standard density, ρ the corresponding length of free path at d_0, formula (1) becomes

$$\delta = \frac{v_0}{V} \frac{\lambda}{\sqrt{\theta_0}} \sqrt{\frac{\theta}{m}} \left(3 + \frac{\lambda}{\phi} \frac{d}{d_0}\right).$$

[1] *Phil. Mag.*, April, 1891.

[2] *Phil. Mag.*, September, 1892.

In view of the imperfect assumptions it will be better not to attach too much weight to the value of the constants, but it may be worth while to examine the more general formula.

$$\delta = \sqrt{\frac{\theta}{m}}\lambda\,(a + b\lambda d)$$

The experiment verification of this formula is attended with considerable difficulty. Nevertheless the results thus far obtained show that it may be considered as a good first approximation to the truth.

The following are the points which seem to be fairly well established:

1. When the pressure is below one-thousandth of an atmosphere, the second term, $b\lambda d$, may be neglected.

2. Under this condition the width of the line is roughly proportional to the square root of the molecular weight. This is shown in the table.

3. The width increases as the temperature rises, the rate being not very different from that of the square root.

4. When the pressure is increased, the width increases in a nearly linear proportion. (This is shown in Fig. 2, in which the ordinates are widths of line in tenth-meters, and the abcissae are pressures of surrounding gas in mm.)

5. The rate of this increase varies considerably with different substances, but in general it is more rapid, the smaller the molecular weight, and while the actual results can scarcely be said to prove that the rate is inversely as the square root of the molecular weight, they do not differ very greatly from this proportionality.

6. At low pressures the proportionality with λ is not proved—there being about as much evidence for an increaseof δ with λ as for a decrease.

7. At high pressures the width increases with wave-length, but the exact law is not determined.

8. The nature of the surrounding gas or vapor is of secondary importance.

In regard to cause 2 it may be remarked that at pressures below one atmosphere the length of the free path is of the order of one hundred times the radius of the sphere of action of the molecules. It appears probable therefore that at such pressures the number of free vibrations is also at least one hundred times as many as those whose period is modified by collision, and the effect of these modified vibrations would be correspondingly small in broadening the spectral lines except at great densities.

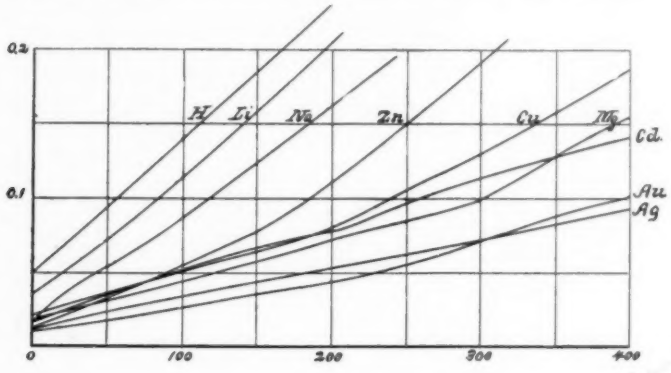

FIG. 2.

It appears probable that the vibratory energy is supplied during "collisions." It does not necessarily follow that the source of this energy is the motion of translation of the molecule (that is, a function of the temperature), but we may suppose, with J. J. Thomson, that there is an interchange of partners among the molecules—which could not occur except on collision—and it seems reasonable to suppose that during this interchange, the chemical (electrical) energy of combination is transformed into electrical vibrations (light).

If we suppose the amplitude of the vibrations to vary as some inverse power of the distance, r, between the approaching molecules we would have

$$\frac{A_0}{A} = \left(\frac{r}{a}\right)^t,$$

A_0 being the amplitude corresponding to the nearest distance a,

But it can be shown that the distance e traveled by the molecule when the amplitude has fallen to one-half its maximum value, in the case of hydrogen at 100 mm. pressure, is of the order of a ten thousandth of a millimeter, so that by the preceding equation $a = (\frac{1}{2})^{\frac{1}{p}} \times e$. a is also of this order of magnitude unless p be small. But this is the order of magnitude of the free path of the molecule, and in all probability the nearest approach is hundreds or thousands of times smaller; so that we are forced to conclude either that p is very small — which is only another way of saying that the amplitude is nearly independent of the distance between the molecules; or else that such a variation of intensity is not an important factor in broadening the spectral lines.

This seems at least conclusive for low or moderate densities. On the other hand, for greater densities, it seems not unlikely that the mutual influence of the molecules may be of great importance.

Where the broadening of the lines is unsymmetrical, and especially where the line is terminated at one side by a sharp edge such broadening can be explained by certain assumptions concerning the law of action of the molecules on their rates and intensities of vibration.

Thus, in the preceding illustration, if

$$\phi(x) = \frac{a}{a^2 + x^2} \cos mx + \frac{x}{a^2 + x^2} \sin mx,$$

the resulting intensity of the spectrum is $I = e^{-4\pi an}$ for positive values of n, and zero for negative values.

If this view is correct, then starting with zero pressure the line should always begin to broaden symmetrically at first, and the assymmetrical broadening should not appear until the pressure is very considerable. As in most cases the width of the spectral lines at low pressure is so small that it is usually masked by imperfections of the spectroscopes employed, this question must be attacked by interference methods.

The method of deducing the form and distribution of light in a spectral line by the visibility curve of its interference fringes

has already given very promising results; but if rigorously interpreted, as shown by Lord Rayleigh,[1] it is limited to the case of symmetrical distributions of light in the source. In fact, for an unsymmetrical source we have

$$V = \sqrt{C^2 + S^2} \tag{1}$$

which gives, however, no information concerning C or S.

In order to determine these another equation is necessary, and this is furnished by the change of phase. If ϕ is the phase of the interference band we have

$$\tan \phi = \frac{S}{C} \tag{2}$$

These two equations determine S and C, and from them we have by Fourier's theorem

$$\sqrt{I} = \psi(n) = \frac{1}{\pi}\int_0^{\infty} d\alpha \cos \alpha n \int_{-\infty}^{+\infty} d\lambda \psi(\lambda) \cos \alpha \lambda$$

$$+ \frac{1}{\pi}\int_0^{\infty} d\alpha \sin \alpha n \int_{-\infty}^{+\infty} d\lambda \psi(\lambda) \sin \alpha \lambda$$

$$\text{or } \psi(n) = \frac{1}{\pi}\int_0^{\infty} C \cos \alpha n \, d\alpha + \frac{1}{\pi}\int_0^{\infty} S \sin \alpha n \, d\alpha.$$

At first it would seem that the determination of the "ϕ" curve would be attended with insurmountable difficulties; for it is practically impossible to divide or even to read a scale so accurately as to find directly the difference of phase, which is, of course, but a fraction of a wave-length.

The following method, judging from some preliminary results obtained in the course of the determination of the length of the meter in light-waves,[2] gives promise of furnishing the required solution.

Suppose an "intermediate standard"[2] consisting of two plane parallel surfaces one centimeter apart to be placed in the "interferometer" and circular fringes obtained for both surfaces.

The phase α and α' of the central dark circle for both surfaces is measured by the "compensator," and the standard is

[1] *Phil. Mag.* November 1892.

[2] Tome XI. *Trav. et Mem. Bur. Int. des Poids et Mésures.*

advanced through its own length and the measurement repeated. If the phase ϕ is constant the difference of phase $a—a'$ will also be constant; if not, then $\phi_n = \Sigma^n_1 (a–a') – n (a–a')_0$ provided the change be not too rapid. If this should be the case, a shorter distance-piece or "intermediate standard" must be used, or two standards of different lengths may be used, and the "ϕ" curve determined by the same process as is employed in thermometer calibrations.

The following experiment appears to prove conclusively that the temperature (velocity of translation of the molecule) in the case of the electrical discharge through rarefied hydrogen, must be remarkably low, though it may be premature to say that it is only a few degrees above that of the surrounding atmosphere.

Dry hydrogen, moderately pure at a pressure of about 1 mm. was contained in a vacuum tube made of hard glass whose section varied gradually from 15 mm. to 1 mm.

On the passage of the spark the characteristic spectrum of hydrogen was observed—the brightness increasing as the diameter diminished.

The light was directed into the interferometer, and after interposing a red glass it was found that the interference fringes (conc. circles) were much more clearly visible at a portion of the tube where the diameter was about 4 mm., and this portion was accordingly employed. The tube was then surrounded by a thin roll of sheet copper, except at the part examined, and this was heated by a Bunsen flame. The falling off in distinctness of the fringes was at once visible, and when a thermometer placed in contact with the copper showed a temperature below 300° C., the distance at which interference was still visible was about three-fourths of its value at the lower temperature. If the actual temperature in the first measurement is 50° or about 320° absolute, then the second temperature (supposing that the thermometer indicated—very roughly of course—the increase in temperature) was about 570°. The corresponding ratio of velocities is about $\sqrt{\frac{320}{570}} = \frac{3}{4}$, which agrees with the result of

experiments. If, on the other hand (as has been assumed by those who believe the temperature to be the efficient cause in producing incandescence), it be supposed that the first temperature was of the order of 7000°, the second could not have been more than 7300° and the corresponding ratio of velocity (and also of difference in path at which interference phenomena are still visible) would be $\sqrt{\frac{7000}{7300}} = .98$, which is so near unity that it would not be possible to detect the change.

Aside from this, the distance at which interference is visible in the case of hydrogen at low pressure is that which corresponds to a velocity of translation of from 2000 to 4000 meters per second; so that even from this measurement alone it would follow that the temperature must be between 0° and 300° C.

Since the pressure of hydrogen vapor in solar prominences (at any rate for the extremity farthest from the Sun's disk) must be extremely low, it follows that the greater part of the width of the corresponding spectral lines must be due chiefly to molecular velocities. Accordingly, an accurate measurement of this width will give the temperature of the prominence.

As the details of the methods by which the preceding experimental results were obtained have not yet been published it may be of interest to describe here the arrangements used for the source of light.

It gives me great pleasure to acknowledge in this connection the valuable assistance of my colleague, Mr. S. W. Stratton.

Many of the experiments were simple modifications of those described in a previous paper and no further notice of them is necessary. For substances whose point of volatilization was higher than the melting point of glass, the following device was employed:

A glass globe *G*, with a funnel-shaped aperture, *F*, closed by a piece of plate glass, is attached to the iron tube *T*, and the enclosed space is connected with a mercury pump, the tube being filled with mercury, forming a barometric column. This permits a long rod to rotate within, thus breaking contact

between the disk *W* and the spring *S*. Either *S* or *W* or both are of the material whose spectrum is to be investigated.

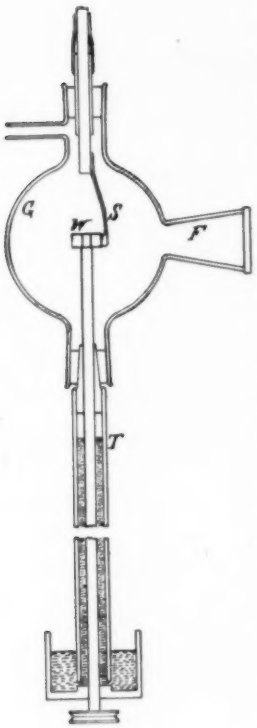

FIG. 3.

The extra-current spark, even with two or three storage cells, and with or without an induction coil, is sufficient to produce a very brilliant light, especially when the pressure within the globe has been reduced to a few millimeters.

Various modifications have been employed, especially one in which the break was produced by a reciprocating instead of a rotary motion. An essential point is to have all the parts readily detachable, so that the globe which rapidly becomes coated with a metallic deposit may occasionally be cleaned.

By constricting the attachment of the funnel at *O* to about 10 mm., the glass plate *P* may remain for days or weeks without cleaning.

It is hoped that future experiments along the lines here indicated may throw new light upon many of the important problems suggested.

THE MODERN SPECTROSCOPE. XIII.

A NEW MULTIPLE TRANSMISSION PRISM OF GREAT RESOLVING POWER.

By F. L. O. WADSWORTH.

IN order to obtain a large resolving power in a prism train it is necessary, with the ordinary arrangement, to considerably increase either the size or the number of prisms in the train. Which of these methods it is best to employ depends on the particular purpose for which the spectroscope is used. Thus in the case of the astronomical spectroscope, which has recently been discussed by the writer,[1] the balance of optical and mechanical advantages seem to lie on the side of a large number of small prisms, and the same considerations would point to this as the proper solution of the problem in the case of most laboratory spectroscopes also. On the other hand, since the angular dispersion of the spectrum is directly proportional to the number of prisms but is independent of their size (the refracting angle remaining constant) it follows that, for photographic work, in which it is necessary or desirable that a large portion of the spectrum should be in the field and in focus at once, the first method is the one to use. The use of large prisms is also absolutely essential in cases where very long slits are necessary, either for securing a spectrum of maximum total intensity, as in bolometric or photometric work,[2] or in the spectroheliograph, where a large image of the solar surface is to be photographed.

Unfortunately for such cases, the limit of size is soon reached, being set in the case of glass by the impossibility at present of

[1] "General Considerations Respecting the Design of Astronomical Spectroscopes," this JOURNAL, January, 1895.

[2] It was shown in the article referred to above, that the total energy in the spectral image depends directly on the height of the illuminated portion of the slit, and in order that great height may be used, a prism of large aperture, at least large vertical aperture, is essential.

properly annealing very large and thick blocks of glass,[1] and in the case of natural substances, such as quartz, rock salt, fluorite, etc., by the scarcity of material; it being an exceptional occurrence to find a large and perfect crystal which can be utilized for the purpose of prism or lens making. The largest regular spectroscope prisms (of refracting angle of 30° or over) of which the writer has knowledge are three which have been constructed by Mr. Brashear within the last few years; one a quartz prism of 30° refracting angle, 12^{cm} high and $13\frac{1}{2}^{cm}$ on the face, for the Harvard College Observatory;[2] a rock salt prism of 60° refracting angle, $18^{cm}.5$ high and 13^{cm} on the face,[3] used in the recent investigations of the infra-red spectrum at the Smithsonian Astro-Physical Observatory; and the third a flint glass prism of the same refracting angle, $16\frac{1}{2}^{cm}$ high and 14^{cm} on the face, now in the possession of the same institution.

To even duplicate one of these would be a matter of considerable difficulty if not an impossibility at present, except in the case of the glass, and even in this case a good deal of expense, to say the least, would be encountered, as more than a year was spent by the makers of the glass before they succeeded in producing a block of sufficient homogeneity for the above prism. Trains of several such prisms are therefore practically unattainable.

In the case of such large prisms, therefore, it becomes a question of considerable importance to determine, first, the form of the prism and, second, the conditions of use which will enable a maximum of resolving power to be obtained with a given quantity of material, or, to put it more specifically, with a block of a certain size. Let r denote the resolving power, D the angu-

[1] The method of building up a compound prism of thin prismatic plates, recently described by the writer (see article "Some New Designs of Combined Grating and Prismatic Spectroscopes of the Fixed Arm Type," this JOURNAL, March, 1895), may perhaps enable larger prisms to be constructed than has heretofore been possible, but here again the question of expense soon sets a limit to unusual increase in size.

[2] Cut from a crystal belonging to the Boston Academy of Natural Science.

[3] Cut from one of the great blocks of rock salt shown in the Russian exhibit at the World's Fair.

U of M

lar dispersion, and ϕ the refracting angle of the prism, n the index of refraction, i the angle of incidence, and θ the angle of minimum deviation.

Then we have

$$r = (t_1 - t_2)\frac{dn}{d\lambda},$$

where t_1 and t_2 are the greatest and least thickness of material traversed by the refracted rays and $\frac{dn}{d\lambda}$ the dispersive power of the material; whence we see that for a simple prism of material of a given dispersive power, the resolution is independent of the refractive angle, and depends only on the length of base of the prism. Thus in Fig. 1 all of the prisms having a common base, a, b, have the same resolution, while the one of largest refracting angle contains by far the least material and could be cut from the smallest block.

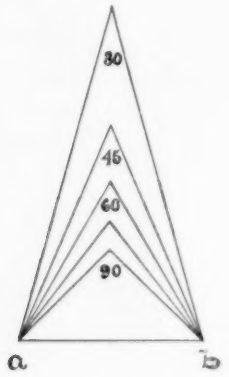

FIG. 1

The use of large refracting angles has also the very decided advantage, in the case of large prisms, of reducing very greatly the dimensions of all of the other parts of the spectroscope, especially the aperture of the collimating and view telescopes. This reduction in aperture is even more rapid than the diminution in volume of the prism, for it depends not only on the length of face, f, which decreases as the refracting angle ϕ increases, but

also upon i which is again a function of ϕ. Thus for unit base, $a, b = 1$ we have

$$l = \frac{1}{2 \sin \frac{\phi}{2}},$$

and for the horizontal aperture or curtate face

$$a = l \cos i = \frac{\cos i}{2 \sin \frac{\phi}{2}} = \tfrac{1}{2} \frac{\sqrt{1 - n^2 \sin^2 \frac{\phi}{2}}}{\sin \frac{\phi}{2}}.$$

The values of v, the volume of a prism of unit base, and of l, the length of face, for different refracting angles from 30° to 82½° are given in Table I, and the values of a for the same angles and for four values of n, *i.e.*, 1.5, 1.6, 1.7 and 1.8, in the third column of Tables II, III, IV and V. We see from these that in the case of the low index $n = 1.5$, the size of the telescopes required for a prism of 80° angle is only about $\frac{1}{10}$ that required for one of 30° of the same resolving power, while in the case of the higher indices the advantage is even more marked.

On the other hand, the increase in the refracting angle increases the loss of light by reflection, and also injures the definition by increasing the effect of defects in the surfaces of the prism. With such prism surfaces, however, as Mr. Brashear produces, the second effect is insignificant and the first need only be considered. If we suppose that the incident beam of light may be considered to be made of two polarized beams of equal intensity, one polarized in a plane perpendicular to the plane of incidence and the other in that plane, and if Δ' and Δ'' represent the respective losses in these two beams by reflection at the first surface, the total loss in the incident beam at the first reflection will be

$$\Delta = \tfrac{1}{2}(\Delta' + \Delta'') = \tfrac{1}{2} \frac{\sin^2 (i - r)}{\sin^2 (i + r)} + \tfrac{1}{2} \frac{\tan^2 (i - r)}{\tan^2 (i + r)}.$$

If the prism is placed at minimum deviation

$$r = \frac{\phi}{2}$$

$$\text{and } i = \sin^{-1} n \sin \frac{\phi}{2}$$

and the expressions for Δ' and Δ'' in terms of ϕ and n alone become

$$\Delta' = \frac{1}{2}\left(\frac{n^2 \cos\phi - 2n \cos\frac{\phi}{2}\sqrt{1 - n^2 \sin^2\frac{\phi}{2}}}{n^2 - 1}\right)^2$$

$$\Delta'' = \frac{1}{2}\Delta'^2\left(\frac{1 - n^2 \tan g^2\frac{\phi}{2}\cos\phi - 2\, n \cos\frac{\phi}{2}\sqrt{1 - n^2 \sin^2\frac{\phi}{2}}}{1 - n^2 \tan^2\frac{\phi}{2}}\right)^2$$

For two faces the loss will evidently be

$$\Delta_2 = 1 - [\tfrac{1}{2}(1 - \Delta')^2 + \tfrac{1}{2}(1 - \Delta'')^2],$$

and for a number of faces, Δ_n, or a number of prisms $= \frac{n}{2}$

$$\Delta_n = 1 - [\tfrac{1}{2}(1 - \Delta')^n + \tfrac{1}{2}(1 - \Delta'')^n].$$

We have finally to consider the question of angular dispersion, for this is also a function of the refracting angle.

The expression for D is

$$D = \frac{d\theta}{d\lambda} = \frac{d\theta}{dn}\frac{dn}{d\lambda} = \frac{r}{a} = \text{Const. } f(\phi),$$

since r is itself independent of ϕ. The quantity $f(\phi) = \frac{1}{a}$ may be termed the dispersion coefficient of the prism.

The values of Δ', Δ'', Δ_1 and Δ_2, and $f(\phi)$ have also been computed for the same range of refracting angle and refractive index as given above for v and a, and will be found in Tables II, III, IV, and V. The relation between ϕ and Δ_2 is also shown in the curves of Fig. 2 and that between ϕ and v, and ϕ and D in the curves of Fig. 3.[1]

[1] In the latter figure the values of the ordinates for curve A, (which shows relation between v and ϕ), have been multiplied by 2, and the ordinates of curves, B, C, D, E, (showing relation between ϕ and D for indices of 1.8, 1.7 1.6 and 1.5 respectively), have been divided by 2 in order to make the relation between v and D more obvious.

TABLE I.

Refracting angle=ϕ	Face of Prism = l	Volume of Prism = v	Refracting angle = ϕ	Face of Prism = l	Volume of Prism= v
30°	1.932	.933	64°	.943	.400
35°	1.663	.793	68°	.894	.371
40°	1.462	.687	72°	.850	.344
45°	1.307	.604	75°	.822	.326
50°	1.183	.536	77½°	.799	.312
55°	1.082	.480	80°	.778	.298
60°	1.000	.433	82½°	.758	.285

TABLE II.

Repeating ang. = ϕ	Angle Incidence = i	Aperture or curtate face a	Δ'	Δ''	$\Delta_1 = \frac{1}{2}(\Delta' + \Delta'')$	Δ_2	Dispersion Coefficient = $\frac{1}{a}$
30°	22° 51′	1.780	.0495	0.0315	0.0405	0.0793	0.5618
35°	26° 49′	1.484	.0537	.0282	.0409	.0801	0.6738
40°	30° 52′	1.255	.0591	.0244	.0417	.0815	0.7970
45°	35° 2′	1.070	.0662	.0200	.0430	.0837	0.9348
50°	39° 20′	.915	.0754	.0158	.0456	.0883	1.093
55°	43° 45′	.782	.0884	.0109	.0497	.0953	1.278
60°	48° 35′	.661	.1057	.0046	.0552	.1047	1.512
64°	52° 39′	.572	.1255	.0012	.0633	.1188	1.747
68°	57° 1′	.487	.1529	.0000	.0765	.1412	2.054
72°	61° 51′	.401	.1937	.0045	.0991	.1795	2.492
75°	65° 57½′	.335	.2401	.0168	.1285	.2280	2.988
77½°	69° 52′	.275	.2974	.0414	.1693	.2938	3.636
80°	74° 37′	.206	.3905	.1000	.2452	.4093	4.846
82½°	81° 30′	.112	.5902	.2965	.4433	.6686	8.922

TABLE III.

30°	24° 28′	1.758	0.0670	0.0410	0.0540	0.0993	0.569
35°	28° 45½′	1.458	.0730	.0363	.0547	0.1060	0.686
40°	33° 11′	1.223	.0812	.0308	.0560	.1083	0.817
45°	37° 45′	1.033	.0918	.0243	.0581	.1116	0.968
50°	42° 33′	.873	.1036	.0171	.0603	.1151	1.145
55°	47° 33′	.731	.1259	.0095	.0677	.1274	1.368
60°	53° 8′	.600	.1566	.0030	.0798	.1473	1.667
64°	57° 59′	.500	.1919	.0000	.0959	.1735	1.999
68°	63° 28′	.399	.2462	.0055	.1258	.2213	2.504
72°	70° 12′	.288	.3426	.0390	.1908	.3222	3.470
75°	76° 55′	.186	.4863	.1392	.3127	.4976	5.378

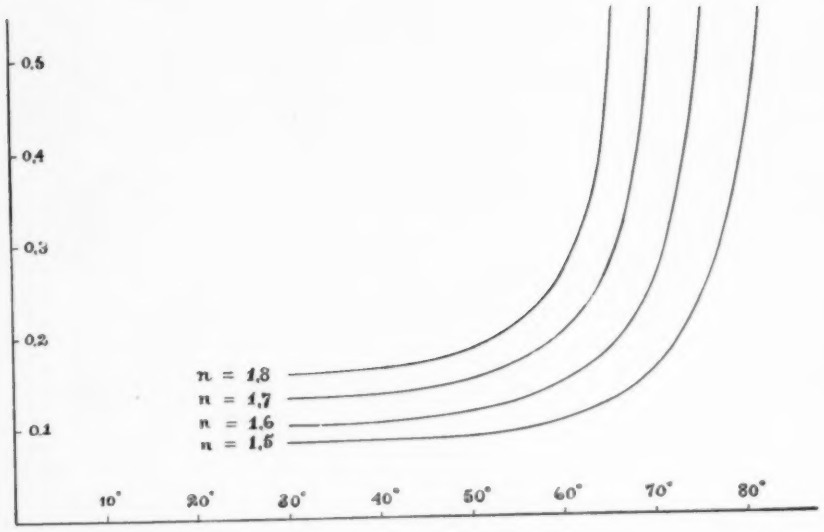
0.5
0.4
0.3
0.2
0.1
n = 1.8
n = 1.7
n = 1.6
n = 1.5
10°
20°
30°
40°
50°
60°
70°
80°

FIG. 2

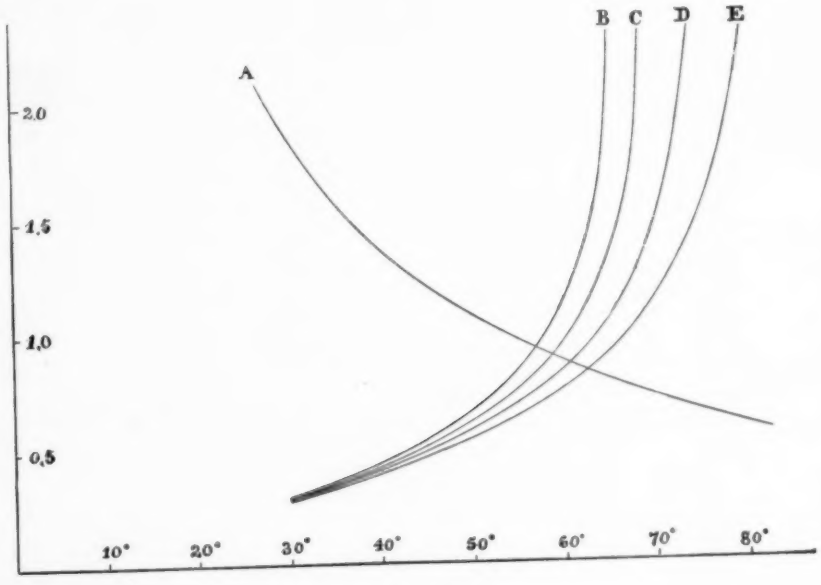
B
C
D
E
A
2.0
1.5
1.0
0.5
10°
20°
30°
40°
50°
60°
70°
80°

FIG. 3

TABLE IV.

Repeating ang. $= \phi$	Angle Incidence $= i$	Aperture or curtate face a	Δ'	Δ''	$\Delta_1 = \frac{1}{2}(\Delta' + \Delta'')$	Δ_2	Dispersion Coefficient $= \frac{1}{a}$
30°	26° 6′	1.735	0.0857	0.0506	0.0682	0.1214	0.5764
35°	30° 44½′	1.429	.0901	0.0441	0.0670	0.1292	0.6998
40°	35° 33′	1.189	.1057	.0364	0.0710	0.1358	0.8408
45°	40° 35′	.992	.1212	.0275	0.0743	0.1416	1.008
50°	45° 55½′	.760	.1422	.0175	0.0799	0.1494	1.315
55°	51° 43′	.671	.1744	.0073	0.0909	0.1665	1.491
60°	58° 13′	.527	.2238	.0003	0.1120	0.1991	1.899
64°	64° 16′	.410	.2885	.0048	0.1467	0.2517	2.441
68°	71° 55′	.277	.4084	.0493	0.2289	0.3731	3.608
72°	88° 28′	.228	.9251	.7983	0.8617	0.9772	4.394

TABLE V.

30°	27° 46′	1.709	0.1059	0.0600	0.0829	0.1585	0.5850
35°	32° 46′	1.398	0.1172	0.0515	0.0843	0.1605	0.7152
40°	38° 0′	1.152	0.1328	0.0412	0.0870	0.1643	0.8680
45°	43° 32′	.900	0.1543	0.0292	0.0917	0.1711	1.111
50°	49° 31½′	.768	0.1855	0.0152	0.1003	0.1834	1.302
55°	56° 13′	.602	0.2337	0.0036	0.1187	0.2100	1.661
60°	64° 10′	.436	0.3177	0.0027	0.1273	0.2700	2.294
64°	72° 32′	.283	0.4507	0.0491	0.2499	0.3971	3.532
66°	78° 38′	.181	0.5914	0.1644	0.3779	0.5675	5.528

An inspection of these tables or better, of the curves, will at once enable us to determine the best value of the refracting angle to use in any given case. Since the main object of using a single large prism rather than several small ones is to reduce the angular dispersion, it follows that if this object is of primary importance we cannot use prisms of refracting angle much exceeding 65° for materials of low refractive index or 55° for materials of high, since beyond this point any small increase in the angle increases greatly the dispersion without appreciably diminishing the volume. Large refracting angles are likewise objectionable on account of the greatly increased loss of light by reflection at the surfaces, although this increase is not quite so rapid as in the case of dispersion. If in lack of a better criterion we choose as a limiting angle that at which a tangent to the curve makes an angle of 45° with the axis, we find from Fig. 2

that this corresponds in the case of $n = 1.5$ to $\phi = ,70°$ and for $n = 1.8$ to $\phi = 58°$.

On the other hand it is readily seen that there is no great gain in reduction of dispersion and almost none in reduction of loss by reflection, by decreasing the angle of the prism below 60°; in fact, in the case of a prism of refractive index of 1.5 the latter loss is diminished by only about 2½ per cent. by decreasing the angle from 60° to 30° while the aperture and dimensions of the other parts of the spectroscope are increased nearly three times.

If the condition of small dispersion is of less consequence then it is also readily seen that the refracting angle may be increased with decided advantage until it reaches the limit imposed by the diminution in the brightness of the image. When there is plenty of light we may even use angles as large as 80°, in which case the telescopes and dimensions of the other parts of the spectroscope are less than one-third as large as for an angle of 60° and less than one-eighth those required for a 30° prism. Or, we may suppose that we have the condition of constant aperture given to compare the relative efficiencies of prisms of different refracting angle. To obtain the same resolving power in this case it is evident that we must increase the number of the prisms of the smaller refracting angle in the ratio of $a_1 : a_2$, where a_1 and a_2 are the apertures of the two prisms, each of unit base and of angles ϕ_1 and ϕ_2 respectively. If this is done it is also evident that the dispersion of the two arrangements will be the same, and we have only to compare the relative loss of light and the relative volume of material. Take for example a prism of index $n = 1.5$, of refracting angle of 76° and of given aperture A. In order to obtain the same resolving power with 60° prisms of the same index and aperture two prisms would be required, since $a_1 : a_2 :: 3.15 : 1.51$. The relative volumes of the one prism of 76° and two prisms of 60° are evidently as 0.65 :0.87 and the relative loss of light by reflection as 0.23 :0.19.[1] The advantage as regards decrease in material and weight and

[1] See Pickering's tables, *Am. Jour.*, 45.

increased simplicity is therefore very decidedly in favor of the single prism of 76°, and the increased loss of light (4 per cent. of total) is insignificant in comparison.

The angle of the prism having been determined upon by a consideration of the relative importance of these conflicting conditions, it next becomes important to consider how this prism (supposing only one at our disposal) may be used so as to still further increase its resolving power. It is evident that this can be done only by employing the principle of multiple transmission, *i. e.*, by passing the light more than once through the prism. So far as the writer is aware no form of spectroscope has so far been proposed in which more than two transmissions through the same prism have been made possible. Of the double transmission form the oldest and most frequently used is an instrument of the Littrow type,[1] which has been shown to be only one variety of a large class of single prism mirror combinations.[2]

A more recent form is that which was quite recently invented and described by Newall,[3] in which all three faces of the prism are polished, and the rays, after a double reflection, enter at the face adjacent to that from which they emerge, and retraverse the prism at an angle of almost 60° to their first passage. This arrangement has the one great advantage of widely separating the incident and the doubly refracted beam, and thus, by allowing separate collimating and view telescopes, avoids the main difficulty with the original Littrow instrument —*i. e.*, a general illumination of the field by reflection from the surfaces of the collimating lens. The instrument, how-

[1] The so-called modified forms of Littrow instrument devised by Young, Lockyer, Browning, Grubb and Hilger are really not double transmission instruments at all, since the rays pass through any one portion of the prism but once. The arrangement in fact is simply equivalent to two or more prisms placed one on top of the other instead of one after the other, and it is evident that, for a given resolving prism, just as many prisms, or rather just as much material, is necessary in the one case as in the other.

[2] "Fixed Arm Spectroscopes." *Phil. Mag.*, October, 1894.

[3] *Astronomy and Astro-Physics*, April, 1894, p. 309.

ever, has one disadvantage which is fatal to accurate spectrometer work—the prism can be placed in a position of minimum deviation for only one particular wave-length. It was while working with a modified form of Littrow spectroscope which has been elsewhere described[1] that a form of multiple transmission spectroscope first suggested itself that was not only free from the disadvantages above alluded to, but which allowed of six transmissions through the same prism, thus making it equivalent in resolving power and dispersion to a train of six prisms of the same size. This form, it is true, involves the use of seven plane reflectors, and the loss of light is therefore somewhat greater than in the case of a simple train. It is, however, not presented as preferable to the train when the latter is possible, but as the only substitute for it when only one prism can be had, either because of the expense, or, what is still more serious, the lack of material. Moreover, the objections to the use of this large number of reflecting surfaces are not so serious as might be imagined, first, because the loss of light from a well-silvered glass reflector or speculum is small as compared with the loss from the faces of the prism itself; second, because the reflectors are so arranged in pairs that displacements of the ray by accidental displacements of the reflectors may be almost wholly eliminated; third, because such reflecting surfaces may now be obtained so perfect that there is no sensible injury to definition of an image, after even a far greater number of reflections than here involved, and, finally, because, the cost of such reflectors is *very* much less — probably not more than one-fourth — the cost of an equal number of prisms of the same quality.

The diagrammatic plan of the arrangement of prism and reflectors is shown in Plate IX, Fig. 4 and also in Fig. 5, in which the movable system has been rotated into the zero position in order to show the method of adjustment of the train. The path of the two extreme rays is shown by dotted lines, Fig. 4, and is from the collimator *A*, through the prism to the mirror *B*, thence to *C*, thence to *D* (if it passed from *B* directly to *D*, the position of the

[1] *Phil. Mag.*, July, 1894.

PLATE IX.

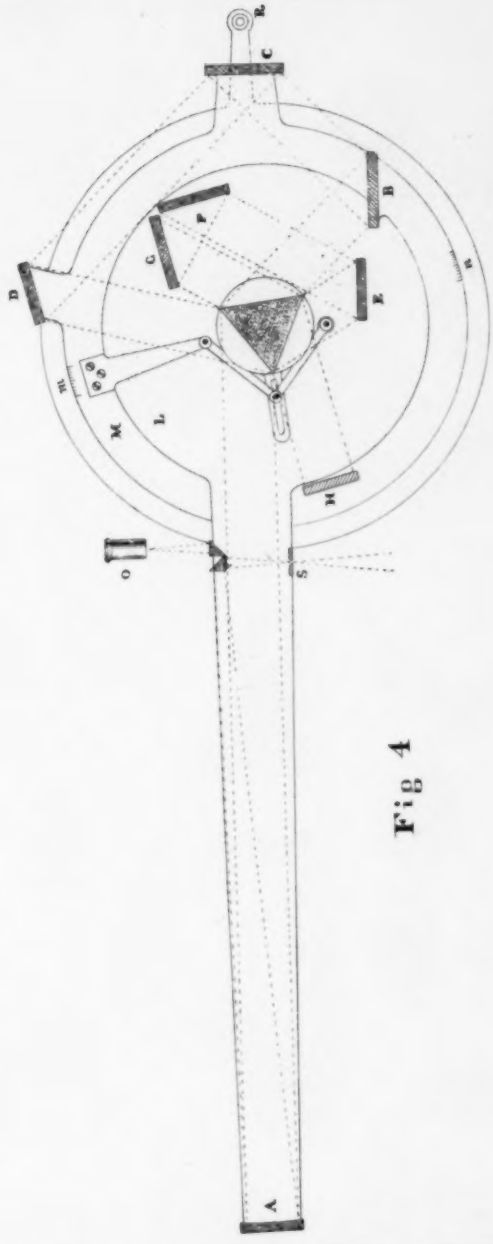

Fig 4

two rays with respect to the refracting edge would be reversed at the second transmission, and the dispersion produced at the first thereby neutralized at the second), thence through the prism again from the first face to the third, thence to the reflectors *E*, *F*, *G*, in succession, thence through the prism a third time, entering at the second and emerging at the third face, in the direction of the arrow.

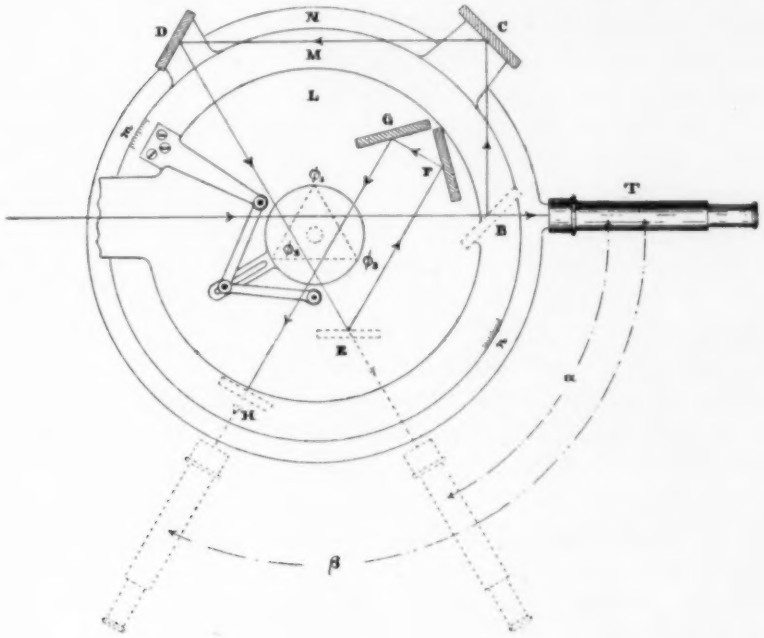

FIG. 5

By placing a seventh reflector at *H*, normal to the emergent ray, the latter may be made to retrace its path, traversing the prism three more times and finally emerging at the first face in the opposite direction to that in which it entered, as in the original Littrow form. The spectrum is formed and observed by any of the methods which have been proposed for this part of the Littrow instrument, best perhaps by the concave mirror

arrangement which has been previously described[1] by the writer, and which is indicated diagrammatically in the drawing. The first set of reflectors, *B*, *C*, *D*, and the final reflector, *H*, are all fixed on the vernier circle, *M*, of the spectroscope and rotate together with it, and the second set of reflectors, *E*, *F*, *G*, are mounted together on an inner table, fixed to the arm which carries the collimator *A*, slit *s*, and observing eyepiece or plate-holder, *o*. The prism itself is mounted on a third table, connected with the outer movable table, *M*, by means of the usual minimum deviation attachment. The outer divided circle, *N*, also rotates and has attached to it an arm, *R*, for a small observing telescope, which is used only in the preliminary adjustment.

It is evident that with this arrangement the ray is transmitted each time at minimum deviation (provided the preliminary adjustments have been properly made), no matter what be the angle of rotation of the vernier circle, *i. e.*, no matter what part of the spectrum be brought to the center of the field. The maximum angle of rotation (or deviation of the first ray), is evidently 60°, but this in the case of a 60° prism corresponds to an index of $n = \sqrt{3} = 1.73+$, which is larger than the index for the shortest wave-lengths of the visible spectrum, in either double-double extra flint or in carbon bisulphide. This angle of rotation, therefore, is amply sufficient for any ordinary prism.

In order to make the preliminary adjustments of this system, the three prism angles ϕ_1 and ϕ_2, and ϕ_3 are first measured by the usual method. The vernier circle is then rotated to the position shown in Fig. 5 and clamped, the prism and first reflector removed and the observing telescope *T* brought into the line of collimation in the usual manner.[2] The mirror *B* is then replaced and *E* is removed, the telescope moved through an angle $a = \frac{1}{2}(\phi_1+\phi_2)$ and the mirror *B* adjusted until the image of the slit again

[1] "An Improved Form of Littrow Spectroscope," *Phil. Mag.*, July, 1894.

[2] Instead of removing the prism the image of the slit may be viewed directly through it by a double refraction and reflection. This would be allowable only when the prism is very nearly equiangular.

falls on the cross wires. Then E is replaced, H removed and the telescope set at the angle $\beta = \phi_2 + \frac{1}{2}(\phi_1 + \phi_3)$, and the slit image again brought to the cross wires by adjusting the mirror E last replaced. Finally the mirror H is replaced and adjusted until the final slit image coincides with the slit itself. The circle is then unclamped and rotated to the position of use and the prism replaced and adjusted to the position of minimum deviation in the usual manner.

It will be noticed that the light which is reflected from the surfaces of the prism falls directly upon the reflectors B, D, E, and G, and would finally, after traversing the prism in the opposite sense, be sent back into the field of the observing eyepiece. To avoid these secondary spectra the prism is tilted very slightly out of the vertical. As the angle of deviation is doubled by reflection an inclination so slight as to be entirely without effect on the transmitted ray will suffice to throw these reflected rays entirely out of the field.

It is evident that by replacing B, E, or H by a telescope we may obtain an ordinary one-transmission, two-transmission or triple transmission train at will. The double transmission train is shown in Fig. 6, in which A is the collimating and T the view telescope, B the reflector which receives the rays after the first transmission through the prism, and P a doubly-reflecting prism which may be substituted for the two reflectors C, D, of Figs. 4 and 5. The reflectors B and P revolve together on the vernier circle of the spectroscope, but the view telescope T remains fixed in position, just as in the Littrow form, although it is unlike this in being quite distinct from, and placed at a considerable angle to, the collimator. It is to be observed that in this form there are no reflections from the prism faces which can reach any part of the field of the view telescope. By placing a fixed reflector at H instead of the view telescope, and placing the latter in the Littrow position, *i. e.*, coincident with the collimator, we obtain a quadruple transmission train which is also free from any troublesome reflections from the prism faces. In Fig. 7 is shown a triple transmission train which is the same in arrange-

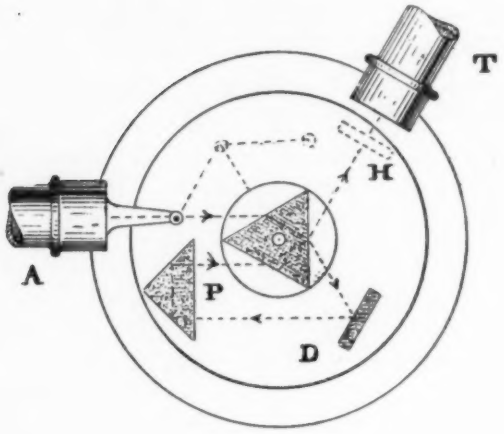

Fig. 6

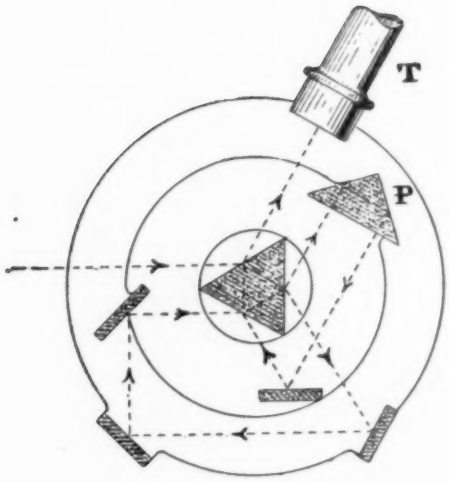

Fig. 7

ment as that first shown in Fig. 5, save that the reflector *H* is replaced by the view telescope, and the two reflectors *F*, *G*, by the prism *P*, and that the order of transmission has been altered, so that after the third transmission the ray emerges from the first face instead of the third as in Fig. 4. This is a convenient arrangement in many respects, for the slit image formed by direct reflection from the first prism face answers admirably for an index. In this case there are secondary reflections from the

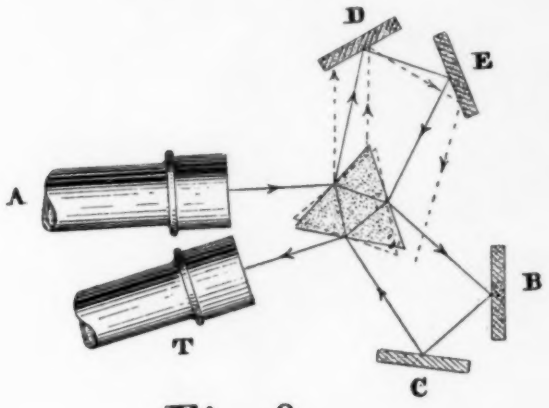

Fig 8

other faces which finally reach the view telescope, but only after so many reflections and transmissions as to be hardly noticeable. They may, as before, be entirely avoided if desired, by tilting the prism faces very slightly out of the vertical.

In case it is desired to work with only one particular wave-length, as in the examination of some individual line in the spectrum, or, as in the use of the spectroheliograph, the number of reflectors may be reduced to two for a double transmission, or four for a triple transmission train by employing the arrangement shown in Fig. 8. Here, of course, the prism must be placed at minimum deviation for the one particular wave-length with which we are concerned, but the index of refraction of the material being known for this wave-length, it is easy to calculate the

proper relative position of the prisms and mirrors in order to secure this result. In Fig. 8, for example, the mirrors are shown in the proper position for transmitting a ray for which the refractive index of the prism is 1.6, which is very nearly the index of the lightest flint glass for the K line of the solar spectrum, the line ordinarily used in spectroheliographic work.

When this arrangement is used for the latter class of work the secondary reflections from the prism faces cannot well be avoided by tilting the latter as in the previous cases, for the field is so wide that too great an inclination would be necessary. It is therefore better to accomplish this object by turning the prism slightly out of minimum deviation, as in Fig. 8. A change of position of 2° changes the angle of the reflected ray by four degrees, or about one part in fourteen, and hence if the distance traversed by the ray between two successive transmissions is fourteen times the aperture, and the prism is turned in such a direction as to diminish the angle of incidence, the light reflected from any one surface will pass entirely outside the next one, as shown by the dotted lines in Fig. 8. This change from the position of minimum deviation will of course also change the angle of deviation of the refracted ray. We know from observation that this change will be small, but in order to determine accurately its amount it is interesting and important to determine the relation between the angle of incidence, i, and the angle of deviation θ.

This will enable us to calculate the necessary small change in the position of the mirrors B, C, D, E, and will also enable us to state how accurately a prism must be adjusted to minimum deviation in order to secure a given degree of accuracy in spectrometric work.

The general expression for the deviation θ' of a prism of angle ϕ is

$$\theta' = i' + i'' - \phi,$$

where i' = angle of incidence on the first face,
and i'' = angle of incidence on the second face of the prism.
If we call δ the angle by which the prism has been turned out of minimum deviation, then

$$i' = i + \delta$$

and expressing the preceding relation in terms of i, δ and ϕ only we obtain

$$\theta' = i + \delta - \phi + \sin^{-1}\left[n \sin\left(\phi - \sin^{-1}\frac{\sin(i+\delta)}{n}\right)\right] \qquad (1)$$

at minimum deviation $\delta = 0$ and the expression reduces to

$$\theta = 2i - \phi \qquad (2)$$

and the problem is to find the difference between (1) and (2) expressed as a function of δ, or

$$\theta' - \theta = \Delta\theta = f(\delta).$$

If δ is a small angle we have, to a high degree of approximation,

$$\sin^{-1}(i+\delta) = \sin^{-1} i + \sin^{-1}\frac{\delta}{\sqrt{1-\left(i+\frac{\delta}{2}\right)^2}}$$

$$= \sin^{-1} i + \sin^{-1}\delta\left[1 + \tfrac{1}{2}\left(i+\frac{\delta}{2}\right)^2 + \tfrac{3}{8}\left(i+\frac{\delta}{2}\right)^4 +\right.$$

Expanding (1) and making use of this general relation and neglecting terms higher than the second order we finally obtain by a series of successive substitutions, which it is unnecessary to develop in detail,

$$\theta' = 2i - \phi + \sin^{-1}\left\{\delta^2\left[1+\frac{\delta^2}{2}\right]\frac{\sin\frac{\phi}{2}\left[n^2 - \left(1+\sqrt{1-n^2\sin^2\frac{\phi}{2}}\right)\right]}{n\cos^2\frac{\phi}{2}\sqrt{1-n^2\sin^2\frac{\phi}{2}}}\right\}$$

whence, since δ is itself small,

$$\Delta\theta = \sin^{-1}[\delta^2 f(\phi, n)].$$

For any given prism $f(\phi, n) =$ constant and since the angle $\sin^{-1}\delta^2$ is nearly equal to δ^2 itself we may say that the relation between $\Delta\theta$ and δ is nearly parabolic.

For a 60° prism the expression $f(\phi, n)$ reduces to

$$f(\phi, n) = \text{Const.} = \tfrac{2}{3}\frac{(n^2 - 1 + \cos i)}{n\cos i},$$

which with the aid of the preceding tables is readily computed for different values of n. We have

for $n = 1.5$ Const. $= 0.395$
for $n = 1.6$ Const. $= 0.667$
for $n = 1.7$ Const. $= 1.015$
for $n = 1.8$ Const. $= 1.525$

The values of $\Delta\theta$ have been computed for values of δ varying from $5'$ to 2° and are given in Table VI.

TABLE VI.

δ	δ^2	$\Delta\theta$			
		$n = 1.5$	$n = 1.6$	$n = 1.7$	$n = 1.8$
$5'$	.00000211	$0''.17$	$0''.3$	$0''.45$	$0''.67$
$10'$	.00000846	$0''.70$	$1''.2$	$1''.8$	$2''.7$
$15'$	.0000190	$1''.57$	$2''.6$	$4''.0$	$6''.0$
$30'$	.0000761	$6''.26$	$10''.5$	$16''.0$	$24''.0$
$45'$	.0001713	$14''.1$	$23''.6$	$36''.0$	$54''.0$
$60'$	.0003045	$25''.0$	$42''.0$	$1'\ 4''$	$1'\ 36''$
$1^\circ\ 30'$	.0006854	$55''.0$	$1'\ 35''$	$2'\ 24''$	
$2^\circ\ 0'$	.001218	$1'\ 40''$	$2'\ 48''$		

A displacement of 2° from the position of minimum deviation in the preceding case therefore changes the direction of the transmitted ray by only about $2\frac{3}{4}'$, a change so small that no readjustment of position of the mirrors is necessary.

The above table also shows at a glance the degree of accuracy required in setting the prism to minimum deviation. If, for example, we desire to make spectrometric measurements to within $10''$ with a prism whose index is 1.6 the prism does not need to be set closer than $30'$ to the position of minimum deviation,[1] or, to put it in another way, the observing telescope may be moved through about 1° ($30'$ on each side of minimum deviation position) without a readjustment of the prism.

UNIVERSITY OF CHICAGO,
September, 1895.

[1] It is perhaps well to call attention to a typographical error at the close of my article in the March number, Vol. I., p. 247, of this JOURNAL. Instead of the deviation from the minimum being $0''.3$ for $\delta = 5'$, as it there appears, it should be $0''.03$ for $\delta = 5'$. The adjustment of the separate prisms to parallelism in this case does not indeed need to be closer than about $15'$ in order that the displacement of the different spectral images shall not exceed the resolving power of even the great Yerkes telescope when used with such a combination of three as is there shown.

CLOSE BINARY SYSTEMS AND THEIR RELATION TO SHORT PERIOD VARIATIONS.

By ALEXANDER W. ROBERTS.

THE discovery of the orbital revolution of δ Cephei, by M. Bélopolsky, is an advance in our knowledge of short period variables of no ordinary importance. Previous to this discovery the movements of β Persei and β Lyrae had been investigated very fully by Pickering, Vogel and Lockyer, and their researches pointed clearly to an intimate connection between the orbital movement and the light variation of the two stars. But the variation of β Persei was considered to be due to eclipse long before the spectroscope gave its testimony to the accuracy of the theory; and as regards β Lyrae it is a variable, *sui generis*, and an investigation of its motion can only in an indirect way assist us to a satisfactory explanation of short period variables—using the term to denote those variables of short period whose light is constantly increasing or decreasing.

With δ Cephei the case is different. It is a good example of a short period variable of constant variation, and any new light or any new discovery bearing directly on the problem of its variation will have a wider application than merely its reference to this particular star. That δ Cephei is a binary star, that it is a binary with a period equal to that of its light variation, are facts which have to do with the large majority of short period variables, for in no important feature is the variation of δ Cephei different from the variation of at least three-fourths of the variables of this class.

So far as my own knowledge of the subject serves me the light curve of δ Cephei given by Professor Schur in the *Astronomische Nachrichten*, No. 3282, may be taken as typical of twelve out of the seventeen southern variables of short period. We have the well known rapid rise to maximum—one of the most striking characteristics of short period variation. There seems

to be no exception to this rule, as an examination of the elements of variation in Chandler's catalogue will testify. With some the rise is exceedingly rapid, as, for instance, in the case of the southern star 3911, η Carinae, where the rise to maximum is less than one-sixth of the time from maximum to minimum; with others again the ratio is one nearly of equality, but with none is the time of decrease less than the increasing period. Then again there is the regular period, and the constancy of amplitude—an amplitude always less than 2.0 mag., oftenest 1.0 or 0.8 mag.—also marked features of short period variation. Only four short period variables seem to show departure from this rule of regular constancy of period and limits of variation, viz.,T Monocerotis, W Virginis, R Triang. Australis, X Cygni.

There are other minor points of family likeness which characterize short period variables, but the two just mentioned are the most salient points.

Now, it is but natural to conclude that some common cause must operate in producing this common type; in the case of δ Cephei its variation is intimately connected with its revolution.

It may be a kind of *per saltum* reasoning to consider all short period variation, with its peculiar characteristics and features, to be due to orbital movement, but to my own mind, immediately on reading of M. Bélopolsky's discovery, the conclusion was clear, inevitable, and the purpose of the present paper is briefly and in a general way to indicate how orbital movement under certain conditions would produce such phenomena as we are familiar with in short period variation.

There are three ways in which revolution would operate in producing changes in the magnitude of one or both members of a binary system:

(1) When the plane of the orbit passes through the Earth we will have eclipse. The eccentricity of the orbit, the ratio of the light of each star per unit of surface, the relative size of the two stars, together with a slight inclination of the plane of the orbit to the line of sight are the chief factors in determining the amount and nature of eclipse at both minima. Theoretically

there is always a second minimum. When the secondary star is a dark body, however, or when the plane of the orbit is slightly inclined to the line of sight, the orbit *very* eccentric and the line of apsides almost coincident with the line of sight, there will be no change at the secondary minimum.

The particulars of variation will vary with each star, but the general type will be the same:—a constant period, a period of rapid rise and fall, another constant period, and then, probably, a second rapid rise and fall, less pronounced than the first.

A simple consideration of the limitations which must always operate in variation of this class will show clearly that while eclipse will explain with partial fullness the variation of the 15 known Argol variables[1] it will not serve as an explanation of short period variation similar to that of δ Cephei. Eclipse may, and probably does, operate in influencing the variation of such variables as η Aquilae and R Sagittae, but it is certainly not the primary cause of their variation.

(2) In the case where a large dark body revolves around a central luminous one, there must be phases, the amount of phase depending on the distance of the stars from one another, the inclination of the orbit, and the reflective qualities of the dark body.

In almost every case, however, these phases would be practically invisible from our system; or rather any increase or diminution in the combined light of both stars would be so small in comparison with the constant light of the primary star as to be imperceptible by the most refined methods of photometric measurement. Under the most favorable circumstances of proximity and light-reflecting qualities of the companion star, it is certain that the change in the light from any system, owing to this cause, would not amount to 0.1 mag. We may, therefore,

[1] Fourteen of these are given in Chandler's Catalogue; the fifteenth is the Southern star X Carinæ, the period of which is 1.083 days. It is no small proof of the care and ability which Mr. Chandler has brought to bear on his Second Catalogue of Variable Stars that he should at once have detected the want of conformity between the Harvard measures and my first elements of this star. The photographic measures are in substantial agreement with the period now given.

leave out of consideration stellar phases as even a minor factor in short period variation.

(3) The third, and to my mind, most effective influence in producing short period variation may more clearly be indicated by referring briefly to the probable orbit of δ Cephei as obtained by Bélopolsky. The form of the orbit is represented in Fig. 1, or rather the projected orbit, for what the inclination of the orbit is we cannot tell.

The ascertained elements are:

Angle between the ascending node and the radius vector	90°
Angle between the ascending node and periastron	88°
Eccentricity	0.514
Projection of the semi-major axis	620,000 miles
Periastron passage	1.05 days after minimum.

The inclination is unknown, but the theory of eclipse would require a value not far from 90°; but such an assumption leads to a value of the mass of δ Cephei that is altogether out of the question.

That δ Cephei has scarcely twice the weight of Jupiter is surely so great an improbability that it makes impossible our acceptance of the supposition on which it rests.

Indeed, the smallest value of the mass of δ Cephei which we could accept, as in harmony with what we know of the mass and light of other systems, would necessitate a value of the inclination considerably less than 45°. Yet another insuperable difficulty in the way of our accepting the eclipse theory of variation is the position of the minimum and maximum phases. The minimum phase on this hypothesis ought to take place at *A* or *P*, Fig. 1: M. Bélopolsky's investigation places it at *m*.

It is unnecessary to point out the very obvious fact that in an orbit such as we have in Fig. 1, however large the component bodies, and however small the orbit, the variation would not be continuous.

There would be comparative constancy when the companion was in quadrature. But such periods have not been noticed by

any observer: all trustworthy observers of δ Cephei bear testimony to the constant waxing or waning of its light.

It is evident, therefore, that the assumption that the inclination is 90° is untenable; and this being so eclipse has no part or lot in the variation of δ Cephei.

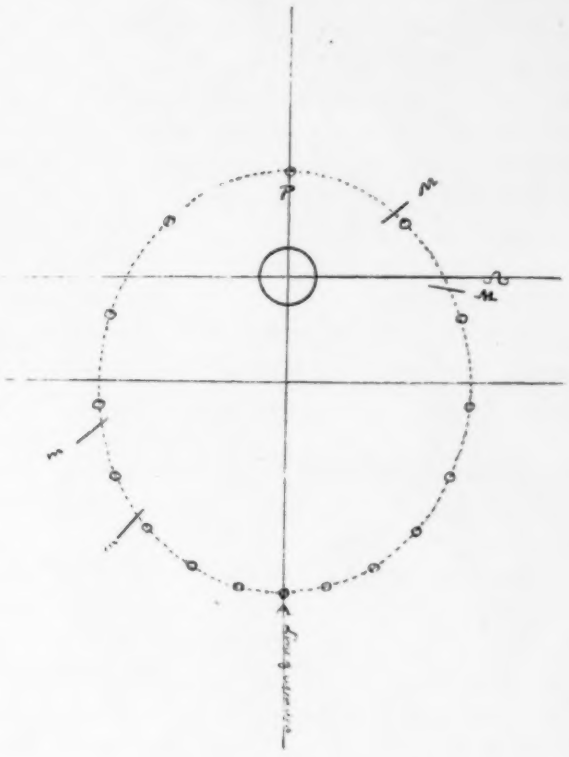

FIG. 1. Orbit of δ Cephei.[1]

The other elements rest on more secure foundation and may, I think, be accepted with confidence.

Now in such an orbit it is evident that at and near apastron,

[1] *M* and *m* are the points on the orbit where, according to M. Bélopolsky, the star passes its maximum and minimum phases. The circular divisions divide the orbit into 16 equal parts as regards time.

the heat which falls upon the companion star will be nine times less than at periastron.

It is but reasonable to suppose that as the companion passes from apastron to periastron some considerable increase in temperature will take place.

The amount of heat transferred from the primary body to the secondary will depend on the capacity for heat of the secondary body. It is also in accordance with the laws of conduction—and we find the laws exemplified each day in the maximum and minimum hours of daily temperature—that the heat of the fainter star will not reach its maximum till *after* the star has passed periastron, the heat continuing to accumulate until the quantity of heat which escapes from the star is greater than that which enters it. And the minimum point will not be reached at apastron passage, but at some point further on, where the amount of inflowing heat equalizes the outflowing. After this point is passed the companion will increase *rapidly*, the rapidity depending on the eccentricity of the orbit, as it is now nearing periastron.

M. Bélopolsky's results put the maximum and minimum points at M and m (Fig. 1).

If the maximum took place at M_1, and the minimum at m_1, it would be more in harmony with the theory I have sketched out here.

Accepting M. Bélopolsky's places, however, it is suggested that the considerable increase of temperature which the companion would necessarily receive on passing periastron P, does not cease to operate in causing change, or, perhaps, that these changes do not attain their full force and vigor till twelve hours after passing P, that is at M.

The star then begins to cool down, the changes in the photosphere become less violent; the emitted light slowly decreases, until at last the star passes the apastron, A.

The increasing heat of the central star is not felt until the companion is a good way on in its return journey. At m the heat begins to tell. The companion brightens up; its light

rapidly increases, until the critical point *M* is again reached, and then the cycle begins anew.

The changes produced by the proximity of the two stars would not be confined to the fainter one alone. There would be action and inter-action, tidal currents, and a different set of conditions, mechanical and chemical, would be in operation at each periastron passage.

This explanation seems to me not only a natural one, but one in keeping with the principal features of short period variation:

(1) A rapid rise to maximum.

(2) Constant variation.

(3) Narrow limits of variation.

It is certainly not a full explanation: it will not meet satisfactorily a variation of say 1.5 mag. or 2.0 mag. unless we suppose, as indeed we are bound to do, that the nearness of the two stars at periastron gives rise to tidal disturbance and consequent increase of temperature. In the case of δ Cephei, if we consider the light of the primary star to be 5.0 and the companion between 7 and 8 mag., then the combined light of both would be 4.9 mag.

At periastron the star would be raised to near the 5 magnitude, giving a combined light of 4.2 mag., a gain of 0.7 mag.

There remains over 0.5 mag. still to be accounted for, and although we might very well claim that this is due to a corresponding change in the light of the primary star, it is, perhaps, as well to leave it as a flaw in the theory.

It is beyond the province of this paper to attempt a full explanation of the theory in its application to variation not in accordance with the general type. There is a departure from the general form of regular light curve so marked, however, that it requires to be dealt with. R Sagittae and η Aquilae may be taken as good examples of this sub-class, of which a secondary maximum is the most striking characteristic. The light curve of η Aquilae, according to Professor Schur, is given in Fig. 2. If now we complete the form of the light curve, so as to make it correspond to the general type, we find that the departure can be

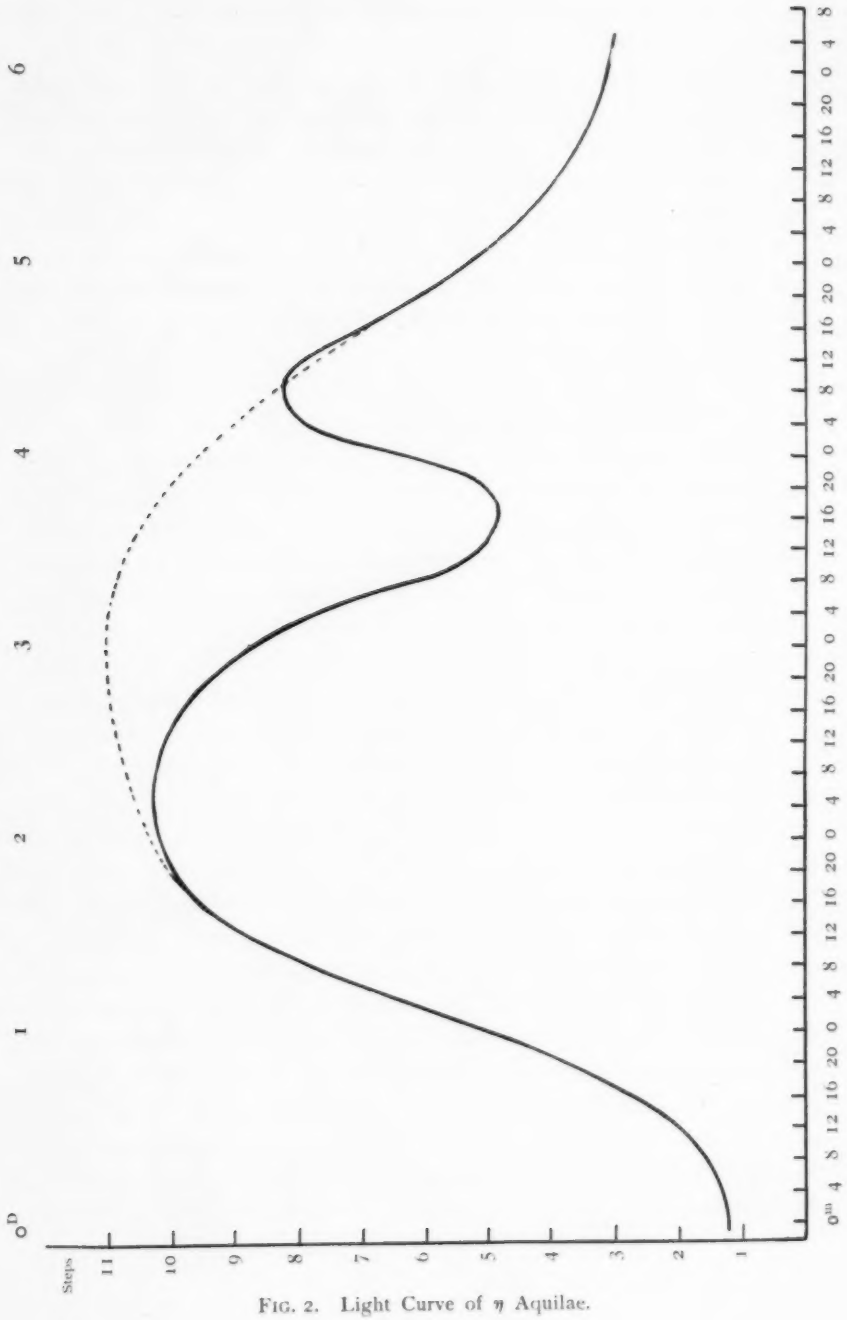

FIG. 2. Light Curve of η Aquilae.

fairly well represented by an eclipse between the periastron and apastron passage.

That is the orbit of η Aquilae is very eccentric and inclined 90° to the line of sight. The eccentricity produces the ordinary type of curve indicated by the dotted line in Fig. 2, and the inclination, by causing eclipse, modifies this general type, the next result being the actual observed curve, indicated by the unbroken line.

There are two tests of the accuracy of the theory thus briefly sketched. One of them is crucial, and can be applied at once; the other is no less final, but time, decades indeed, must pass before a definite answer can be obtained. If the increased light of δ Cephei be due to an increase of temperature, consequent on the proximity of the two stars, it is evident that after the companion star passes periastron, and moves rapidly into quadrature, the displacement of *both stars* should be visible in some of the more powerful spectroscopes—that is assuming M. Bélopolsky's values of the angle between the major axis and line of sight, and the time of periastron passage to be correct.

The second test is one which requires time for its application, and apart altogether from its relation to stellar variation, it is an astronomical problem of great interest.

Does such a binary star as α Centauri vary? At periastron the two component stars, each as large as the Sun, one of them many times brighter than the other, are a little over three times nearer than at apastron. The heat which the less luminous body receives in the former position is ten times greater than that which it receives in the latter. Was, therefore, α Centauri brighter in 1875 than it will be in 1915?

I do not think either eye estimates or photometric estimates will yield a satisfactory answer to this question. To answer it with anything like definiteness and certainty, the measures made at any one time ought to be directly comparable with those taken at another time. Now photography permits of such a comparison, in the case where the disparity between the two component stars is not very great. Photographic images of α_2 α_1 Centauri

have been secured at the Cape Observatory, so well defined, that it is possible to make a very valuable and accurate comparison between the size of their disks. And these photographs are a permanent record, a record which can be immediately and accurately compared with a similar set taken say forty years hence. Now, if such a comparison be made, and if from an examination of the plates we find that the relation between the disks is still the same, 1 : 3 or 1 : 4, as the case may be, then eccentricity has no effect upon magnitude. My own expectation, unwarranted by any definite data, and based on an unproved hypothesis, is that there will be change in the relative sizes of the two disks.

On the relation of short period variation to that of long period, I do not enter; that there is a close relation is certain, and if it can be demonstrated that this class of variation is in whole or in part due to orbital movement, then by thus unifying two distinct divisions of stellar astronomy, variable and double stars, an advance of no ordinary importance will be made in our knowledge of sidereal physics.

LOVEDALE, S. AFRICA,
August 22, 1895.

PHOTOMETRY OF A LUNAR ECLIPSE.

By FRANK W. VERY.

IN order to determine the brightness of the Moon during the total eclipse of September 3, 1895, I used a special photometer designed for the comparison of equal angular areas of two luminous bodies, and permitting the measurement of a wide range of luminosity. The instrument, constructed by Brashear from the author's drawing, consists of an opaque blackened metal diaphragm limiting the field of view of a telescope to a small rectangular aperature divided into halves by a total-reflecting prism, and viewed by a positive eyepiece. The total-reflecting prism is placed behind the diaphragm, that is, on the side toward the objective, and receives light from a comparison-flame through a side tube carrying the apparatus for diminishing the lamp-light. The reduction of the standard light is effected by interposing a series of tinted glasses, carried by a bar which can be pointed in any direction, and clamped when the light of the lamp-flame, seen through a 1^{cm} circular aperture, is central, and the pencil of rays directed parallel with the axis of the bar. A small mirror, silvered on the rear face, and situated at the mouth of the side tube of the telescope at its junction with the swiveling-bar, is then inclined until the light is directed centrally upon the small total-reflecting prism already mentioned, when there are seen, at the center of the field of view, two small luminous patches, each $2^{mm}.5$ square, in juxtaposition, one from the Moon or other luminous body seen through the telescope, the other from the variously diminished lamp-flame. The tinted glasses are carried by brass holders rotating about a cylindrical axis, and, when interposed, rest upon the bar which carries the 1^{cm} circular aperture. There are four pieces of light-blue cobalt glass, "bl.," four pieces of light neutral tinted glass, "ln.," and four of dark neutral tinted glass, "dn.," besides a bundle of a dozen or more pieces of clear, slightly greenish glass, "cl."

The lunar image was formed by a simple lens of $11^{cm}.9$ aperture, and 755^{cm}. focal length, used as a horizontal telescope with the siderostat.

The comparison-light was from a student-lamp, burning a variety of rectified petroleum known as "Elaine," with the flame (about 6^{cm} high) limited by an aperture $1^{cm}.5$ high, so that only the brighest part could be seen.

In determining the absorption of light by the standard glasses, a second student-lamp with nicol-prism polarizer and analyzer, the latter provided with a circle divided to half-degrees, was used in place of the lunar telescope. The comparison-flame with the polarizing apparatus was placed nearer than the other flame in order that the analyzer might be in its most sensitive position, it being difficult to match lights accurately when the principal sections of polarizer and analyzer are less than 45° apart.

The four dark neutral tinted glasses were of the same lot and agreed almost perfectly in their absorption. The same was true for the blue glass, and for all but one of the light neutral tinted glasses. In preparing an apparatus of this sort, a sufficient number of duplicate glasses should be provided, as it is almost impossible to match any that may become broken subsequently. The square of the cosine of the angle between the principal sections of the crossed nicols measures the light, and is given for the principal components of the absorbing train.

Light of undiminished flame	Light through 1st dn.	Light through 2d dn.	Light through 3d ln.	Light through 4th ln.	Light through aberrant 2d ln.	Light through 1 bl.	Light through 10 cl.	Light of undiminished flame
.4494	.0326	.0218	.0996	.0986	.0418	.2871	.1564	.4046
.4564	.0245	.0284	.0924	.0741	.0384	.2350	.1602	.4028
.4408	.0218	.0320	.0826	.0760	.0358	.2470	.1308	.4184
.3875	.0239	.0250	.0945	.0778	.0284	.1963	.1590	.4477
.4512	.0245	.0245	.0807	.1018	.0295	.2887	.1427	.4132
.5139	.0377	.0380	.0610	.0826	.0351	.2365	.1502	.4685
.3555	.0345	.0314	.1025	.0855	.0499	.2218	.1215	.3926
.3405	.0224	.0377	.0679	.0945	.0398	.2545	.1590	.4063
.5297	.0320	.0284	.1114	.1028	.0483	.2545	.1440	.4390
.4200	.0320	.0290	.0845	.0855	.0432	.2380	.1181	.4339
.4345	.0286	.0296	.0877	.0879	.0390	.2459	.1442	.4227

Taking the mean of the readings for the undiminished flame as 0.429, the transmission of light by the standard glasses is:

Dark neutral tint,	1st specimen,	6.67%
" " "	2d "	6.90
	Mean,	6.79% = "*dn.*"
Light neutral tint,	1st specimen,	20.44%
" " "	2d "	20.49
	Mean,	20.47% = "*ln.*"

Aberrant light neutral tint (No. 2),	9.09 per cent. =	"ln_a"
Light blue cobalt glass,	57.32 per cent. =	"*bl*"
Clear (greenish) glass — 10 pieces,	33.61 per cent. =	10 "*cl.*"

As shown by Prof. E. C. Pickering's "Application of Fresnel's Formula" (*Proc. Am. Acad. of Arts and Sci.*, **9**, October, 1873), the transmission of a single plate should be $t = 83.5$ per cent. the transmission by ten plates being 33.6 per cent. if the loss were entirely due to reflection, the formula being

$$t = \frac{1 - r}{1 + (m-1)r},$$

where t is the transmission, r the reflection from one surface, and m the number of surfaces. But the usual loss by reflection from colorless glass is nearer 8.8 per cent. for one plate, and 49.4 per cent. for ten plates, giving $t_{10} = .506$. The observed transmission being $t_{10} = .336$, the loss by absorption in the substance of the clear greenish glass must have been $(.506 - .336) \div .506 = .336$, and the corresponding transmission, $t_{10} = 0.664$, for which the exponential law, $t_n^{\,1} = t_1^{\,n}$ (t_1 and t_n being transmissions by one and by n plates respectively) ought to hold rigidly, giving 4.0 per cent. as the true absorption of a single plate.

Treating the losses separately, we have then:

Loss by absorbtion - -	=	4.0 %	for one plate.			
" " reflection -	=	8.8 "	"	"	"	
Total loss - - -	=	12.8 "	"	"	"	
Resulting transmission	=	87.2 "	"	"	"	"*cl*"

The assumption that the loss of light is entirely due to reflection would make $t_1 = 83.5$ per cent., and the assumption that the entire loss is absorptive gives $t_1 = 89.7$ per cent. The latter differs so little from what is probably the correct result, $t_1 = 87.2$ per cent., that it may be adopted in the present instance, where great accuracy is not needed. The loss of light in the passage through the dark glasses is due almost wholly to absorption. Accordingly we have the following logarithmic factors which, by addition, give the logarithm of the transmission for any combination of glasses:

1 *cl*	9.9526	1 *bl*	9.7583	1 *dn*	8.8319
2 *cl*	9.9053	2 *bl*	9.5166	2 *dn*	7.6637
3 *cl*	9.8579	3 *bl*	9.2749	3 *dn*	6.4956
4 *cl*	9.8106			4 *dn*	5.3275
5 *cl*	9.7632	1 *ln*	9.3111		
6 *cl*	9.7159	2 *ln*	8.6223		
7 *cl*	9.6685	3 *ln*	7.9334		
8 *cl*	9.6212				
9 *cl*	9.5738	ln_2	8.9586		
10 *cl*	9.5265				

JOURNAL OF OBSERVATIONS.

September 2, 1895, one day before the eclipse.

(1) 3 bl. in path of lamp rays. Lamp a little too bright, a little too blue. 2 bl. not blue enough.

(2) 3 bl.+1 ln. too dark. (1) and (2) in faintly purplish *maria*.

(3) 3 bl. lamplight reduced to the right intensity but hardly blue enough to match the brightest streaks. In lucid region south of center of lunar disk. Sky somewhat smoky.

September 3, 1895. J. H. watches for clouds.

(4) In lucid region S. of center. 3 bl.+1 ln. a fair match, but dense clouds of smoke over the Moon.

(5) Same region. Smoke has passed away. 3 bl. an exact match.

(6) Same region. 3 bl.+10 cl. too dark and too green.

(7) In Oceanus Procellarum 3 bl.+9 cl. equal intensity, too green.

(8) Same. 3 bl.+ 6 cl. a little too bright.

(9) Same. 3 bl.+ 11 cl. a little too dark. A pretty good sky.

(10) Another part of Oceanus Procellarum 3 bl. + 6 cl. equal.

(11) Another part of same near the E. limb, 2 bl. + 1 ln. intensity nearly right, too yellow.

(12) 3 bl. + 1 ln., too dark.

(13) 3 bl. + 5 cl. a fair match. $9^h\ 48^m$ Eastern M.T., Almanac time of entrance on penumbra.

(14) Bright S. limb, entirely outside of penumbra, 2 bl. + 4 cl. intensity equal, a little too yellow.

(15) In Oceanus Procellarum not quite halfway through calculated penumbra, 3 bl. + 9 cl. equal. Occasional fracto-cumulus clouds, soon passing away. Sky between clouds quite clear. Distant lightning, S.W.

(16) Dark N.E. limb, near Harding, immersed in penumbra to about 0.7 of its calculated width. 3 bl. + 1 ln. a good match.

(17) Same. 3 bl. + 1 ln. not dark enough. The penumbra has darkened appreciably after an interval of two minutes.

(18) Same 3 min. later. 3 bl. + 1 ln. + 5 cl. a fair match.

(19) Dark limb N. 50° E., near Lavoisier, immersed in penumbra to about 0.9 of its calculated width. 3 bl. + 1 ln. + 5 cl. a fair match, a little too blue.

(20) Same, five minutes later. 2 bl. + 1 ln. + $\ln_2$. a little too dark.

(21) Same, two minutes later. Same glasses a little too bright and too yellow. First contact of shadow at 11^h Eastern M.T.

(22) Same region on the edge of the shadow. 2 bl. + 2 ln. + $\ln_2$. a match.

(23) Same region in shadow. 3 bl. + 3 ln. + $\ln_2$. a little too dark.

(24) Same. 3 bl. + 2 ln. + $\ln_2$. too bright.

(25) Same. 2 bl. + 3 ln. + $\ln_2$. right intensity, too yellow.

(26) Same. 3 bl. + 3 ln. + $\ln_2$. a trifle too bright.

(27) In Oceanus Procellarum a little E. of center. 3 bl. +3 dn. a good match.

(28) Same, but progressively deeper in shadow. 3 bl. + 1 ln. + 3 dn. a good match.

(29) Same, thirteen minutes later. Same glasses a good match. Commencement of totality at $12^h\ 6^m$ Eastern M.T. Appearance of Moon seen through a small telescope, reddish purple, with blue-gray border on the side where the light is disappearing. To the naked eye, the Moon has

more of a coppery tinge, the limb which has just passed into shadow brighter, and blue, like a bright line. From a recollection of previous eclipses, this was fancied to be darker than the average. The present measures, however, may be trusted to give a more reliable estimate than such impressions. Some time was lost at this point in detecting the source of some excessively faint stray light which entered the apparatus.

(30) At the center of the lunar image, just before mid-eclipse 3 bl. + 3 ln. $+ln_2$. + 4 dn., the entire series of darkening glasses, not quite dark enough.

(31) Same region, thirty-five minutes later. The Moon having brightened a little, the same glasses are a fair match.

(32) Same region, twelve minutes later. Same glasses match.

(33) East limb, decidedly brighter as the end of totality approaches. 3 bl. + 3 dn. equal. Totality over at $13^h\ 47^m.5$ Eastern M.T.

(34) East of center, still in shadow. 3 bl. + 3 dn. a match.

(35) In shadow near edge of luminous segment. 3 bl. + 2 dn. too bright.

(36) Same. 3 bl. + 1 ln. + 2 dn. too dark.

(37) In penumbra, just outside of shadow. 3 bl. + 1 dn. too bright.

(38) Same, 3 bl. + 1 ln. + 1 dn. too dark.

(39) Bright N.E. limb, extreme edge of lucid region, and immersed about 0.65 of the calculated penumbral width. 3 bl. + 3 cl. intensity equal.

(40) Same, immersed to 0.45 of the calculated penumbral width. 3 bl. + 2 cl. a match. The illuminated limb looks white with the slightest roseate tint. By contrast the lamp-flame, through three light-blue glasses, looks clear white, but yellow through only 2 bl. and much too bright. At this point thin cirrus clouds formed about the Moon, preventing further measures.

In the 6th column of the following table containing the reductions of the light to zenith and full Moon, $>>$ denotes that the reduced comparison-lamp-light is notably too bright, $>$ that it is a little too bright, $=$ signifies a good match, $<$ a little too faint, $<<$ notably too faint. Seidel's table for atmospheric absorption of light has been used in reducing to zenith.

No.	Eastern M T.	Appt. Zen. D.	Relative Air Mass	Light in Terms of Std.	Character of Match	Estimated exact match	Light reduced to zenith	L. in Terms of Zenithal Full Moon	Radial Position in Shadow	No.
1	Sept. 2			$1.883(10)^{-1}$	* >					1
2	$12^h\ 20^m$	55° 05′	1.762	$3.855(10)^{-2}$	< <	$1.3(10)^{-1}$	$1.52(10)^{-1}$	$5.85(10)^{-1}$	Dark region	2
3	Sept. 3			$1.883(10)^{-1}$	=	$1.9(10)^{-1}$	$2.22(10)^{-1}$	$8.54(10)^{-1}$	Lucid region S. of center	3
4	$9^h\ 15^m$	64 28	2.319	$3.855(10)^{-2}$	=(†)	Rej.				4
5	9 20	63 36	2.250	$1.883(10)^{-1}$	=	$1.9(10)^{-1}$	$2.54(10)^{-1}$	$9.77(10)^{-1}$	" " " "	5
6				$6.330(10)^{-2}$	< <					6
7	9 30	62 05	2.136	$7.058(10)^{-2}$	=					7
8				$9.790(10)^{-2}$	>	$7.4(10)^{-2}$	$9.59(10)^{-2}$	$3.69(10)^{-1}$	Dark reg. Oceanus Proc.	8
9				$5.676(10)^{-2}$	<					9
10	9 39	60 49	2.051	$9.790(10)^{-2}$	=	$9.8(10)^{-2}$	$1.24(10)^{-1}$	$4.77(10)^{-1}$	" " " "	10
11				$6.725(10)^{-2}$	=					11
12	9 44	60 05	2.005	$3.855(10)^{-2}$	< <	$8.8(10)^{-2}$	$1.10(10)^{-1}$	$4.23(10)^{-1}$	" " " "	12
13	9 46			$1.092(10)^{-1}$	=					13
14	10 30	54 51	1.732	$2.124(10)^{-1}$	=	$2.1(10)^{-1}$	$2.45(10)^{-1}$	$9.42(10)^{-1}$	Bright S. limb beyond pen.	14
15	10 35	54 22	1.716	$7.058(10)^{-2}$	=	$7.1(10)^{-2}$	$8.22(10)^{-2}$	$3.16(10)^{-1}$	½ through penumbra	15
16	10 40	53 54	1.697	$3.855(10)^{-2}$	=	$3.9(10)^{-2}$	$4.49(10)^{-2}$	$1.73(10)^{-1}$	.30 pen. beyond shadow	16
17	10 42			$3.855(10)^{-2}$	> >				(15) to (26) in dark regions	17
18	10 45			$2.235(10)^{-2}$	=	$2.2(10)^{-2}$			.25 pen. beyond shadow	18
19	10 50	52 59	1.661	$2.235(10)^{-2}$	=	$2.2(10)^{-2}$	$.51(10)^{-3}$	$9.65(10)^{-2}$		19
20	10 55			$6.089(10)^{-3}$	<	$9.\ (10)^{-3}$	$.0\ (10)^{-3}$	$3.85(10)^{-2}$	.10 pen. beyond shadow	20
21	10 57			$6.089(10)^{-3}$	>	$3.\ (10)^{-2}$	$2.4\ (10)^{-5}$	$1.31(10)^{-2}$		21
22	11 00	52 06	1.628	$1.252(10)^{-3}$	=	$1.3(10)^{-4}$	$.47(10)^{-4}$	$5.65(10)^{-3}$	Edge of shadow	22
23	11 10	51 17	1.599	$1.469(10)^{-4}$	<	$3.4(10)^{-4}$	$.82(10)^{-3}$	$1.47(10)^{-3}$	Shadow at .95 rad.	23
24	11 15			$7.175(10)^{-4}$	> >					24
25	11 20	50 36	1.576	$2.562(10)^{-4}$	=	$2.6(10)^{-4}$	$2.90(10^{-4}$	$1.12(10)^{-3}$	" " .85 "	25
26	11 35	49 48	1.550	$1.469(10)^{-4}$	>	$1.\ (10)^{-5}$	$2.11(10)^{-4}$	$4.27(10)^{-4}$	" " .80 "	26
27	11 50	49 12	1.530	$5.895(10)^{-5}$	=	$5.9(10)^{-5}$	$6.50(10)^{-5}$	$2.50(10)^{-4}$	" " .74 "	27
28	11 52	49 11	1.529	$1.207(10)^{-5}$	=	$1.2(10)^{-5}$	$1.32(10)^{-5}$	$5.08(10)^{-5}$		28
29	12 05	48 54	1.521	$1.207(10)^{-5}$	=	$1.2(10)^{-3}$	$1.32(10)^{-5}$	$5.08(10)^{-5}$	" " .58 "	29
30	12 50	49 11	1.529	$3.122(10)^{-9}$	> >	$1.\ (10)^{-9}$	$1.10(10)^{-9}$	$4.2\ (10)^{-9}$	" " .21 "	30
31	13 25	51 12	1.596	$3.122(10)^{-9}$	=	$3.\ (10)^{-9}$	$3.36(10)^{-9}$	$1.29(10)^{-8}$	" " .37 "	31
32	13 37	52 11	1.631	$3.122(10)^{-9}$	=	$3.\ (10)^{-9}$	$3.40(10)^{-9}$	$1.31(10)^{-8}$	" " .49 "	32
33	13 38	52 16	1.635	$5.895(10)^{-5}$	=	$5.9(10)^{-5}$	$6.68(10)^{-5}$	$2.57(10)^{-4}$	" " .81 "	33
34	14 05	54 45	1.733	$5.895(10)^{-5}$	=	$5.9(10)^{-5}$	$6.87(10)^{-5}$	$2.64(10)^{-4}$	" " .83 "	34
35	14 12	55 24	1.762	$8.682(10)^{-4}$	> >	$5.2(10)^{-4}$	$6.10(10)^{-4}$	$2.35(10)^{-3}$	" " .92 "	35
36				$1.777(10)^{-4}$	< <					36
37	14 15	55 44	1.777	$1.279(10)^{-2}$	> >	$7.7(10)^{-3}$	$9.06(10)^{-3}$	$3.48(10)^{-2}$	.10 pen. beyond shadow	37
38				$2.618(10)^{-3}$	< <					38
39	14 18	56 00	1.789	$1.358(10)^{-1}$	=	$1.4(10)^{-1}$	$1.65(10)^{-1}$	$6.35(10)^{-1}$	.35 " " "	39
40	14 24	56 38	1.818	$1.514(10)^{-1}$	=	$1.5(10)^{-1}$	$1.78(10)^{-1}$	$6.85(10)^{-1}$	.55 " " "	40
									(39) and (40) in lucid reg'n	

* Lamp. † Smoky.

In this table Nos. 4 and 5 show a variation of light in the proportion of 1 to 5 owing to smoke clouds. Fortunately this did not occur again during these measures. Before the eclipse began, photometric measures of fair samples of dark and bright lunar regions showed the lucid regions (0.977 of full Moon light) to be more than twice as bright as the darker parts of the

maria (0.423 of the lucid full Moon). Nearly the same ratio is maintained in such comparisons as Nos. 15 and 40, the former, 0.316 of full uneclipsed Moon light, being in a dark region half covered by the penumbra, the latter, 0.685 of full Moon light, in a lucid region similarly obscured.

The light diminished rather uniformly during the passage of the penumbra, being gradually reduced to about 0.5 per cent. at the edge of the true shadow, but fell off at a more rapid rate in the outer part of the shadow, becoming at 0.6 of the shadow-radius scarcely more than 1 per cent. of the previous remnant at the margin of the shadow, while of the feeble light at 0.6 of the shadow-radius less than 0.5 per cent. remained at 0.4 radius, which seem to be the limit of a more uniformly illuminated area.

The following concise tabular view illustrates the nature of the change better than a curve:

Position of Point Measured Relatively to Eclipsed Area.	Fraction of Full Moon Light Remaining.
½ hour before beginning of eclipse.....	.977
Beyond outer margin of penumbra.....	.942
.50 width of penumbra beyond shadow....	.316
.30 " " " "	.173
.15 " " " "	.096 5
.10 " " " "	.038 5
.08 " " " "	.013 1
Edge of Shadow.....	.005 65
.95 of radius from center of shadow.....	.001 47
.85 " " " "	.001 12
.80 " " " "	.000 427
.74 " " " "	.000 250
.58 " " " "	.000 050 8
.21 " " " "	.000 000 004 2
.37 " " " "	.000 000 012 9
.49 " " " "	.000 000 013 1
.81 " " " "	.000 257
.83 " " " "	.000 264
.92 " " " "	.002 35
.10 width of penumbra beyond shadow....	.034 8
.55 " " " "	.635
.55 " " " "	.685

At mid-eclipse the center of the lunar disk was so situated that the true position of the Sun's limb must have been 31′ from

the nearest limb of the Earth; but atmospheric refraction being twice 35′ for rays which graze the Earth's surface, the atmospheric annulus still continues to transmit and refract into the shadow a portion of sunlight which may be roughly estimated as follows:

The Sun's undiminished light being taken as unity, its light, after transmission by one atmosphere of such quality as would be considered clear in Pittsburg, is diminished to about 0.6, and after passing the thickness equivalent to two atmospheres, 0.36 remain. This last being for an altitude of 30°, I compared with it the light of the Sun when at an altitude of 37′ (relative air-mass=24.4, the zenithal air-mass being unity). The Sun was of a bright red color, but the tint was not so intense a red as on some occasions of very smoky or hazy sky.

The result showed an intensity of

.0000250 at $\lambda = 0.^{\mu}610$ in the orange of the spectrum,
.0000065 at $\lambda = 0.^{\mu}520$ in the green of the spectrum,

relatively to the same light at altitude 30°, or allowing this to have a value of 0.36, and comparing with unabsorbed sunlight:

Sunset light of $\lambda = 0.^{\mu}610$ had an intensity = 0.0000090,
Sunset light of $\lambda = 0.^{\mu}520$ had an intensity = 0.0000023.

A mean sunset light from the entire terrestrial atmospheric annulus, presented at right angles to the Sun's direction, comes from winter as well as summer skies, and will not be as red as the richest tropical sunsets. A mean for the orange and green rays, or 0.00000565, may be taken as the ratio of mean sunset light to undiminished sunlight, although I am aware that there may be wide variations in the numerical value of this quantity. For a vertical passage through air of similar quality, this corresponds approximately to a transmission of

$$(0.00000565)^{\frac{1}{24.4}} = 0.609$$

While the rays refracted to the Moon through mid-width of the atmospheric annulus must traverse a longer path through the air than the observed sunset rays in about the proportion 1.4 to 1.0. allowance must also be made for the greater transmissibil-

ity of the upper air. Thus assuming the transmission to be 0.7, instead of 0.6, for unit depth, about four times as much light should reach the Moon as that which comes to the Earth's surface at sunset, according to the observation already given. On the other hand, any cloud or mist in the lower air must largely diminish the transmitted light. The greater transparency of the upper air evidently tends, in some unknown degree, to compensate for the greater length of the path.

However complex the law of refraction by the Earth's atmosphere may be, the effect of the combined refractions by the atmospheric layers in illuminating the Moon during a total eclipse may be duplicated by that of an equivalent homogeneous annular prism. The refraction, however, is strictly not that of a single fixed prismatic annulus, but rays from different parts of the Sun are refracted by differently placed annuli, of similar refracting section, in such a way as to reach all parts of the shadow; and on account of the angular area of the Sun, the red light which has passed through the deeper layers of air is not confined to a particular zone in the shadow as it would be if the luminous source were a point.

The width of the atmospheric annulus which becomes visible by its refraction of the Sun's light, can never equal the entire depth of the Earth's atmosphere. Within the shadow, near to one edge, light is received which has been slightly refracted through thin upper air at the corresponding limb of the Earth; but light also reaches this point by refraction through the denser air of lower layers at the opposite limb. No light which has been refracted by the denser air can reach the corresponding edge of the shadow, but such light must be bent in to the center or to the opposite edge of the shadow.

As shown subsequently, it is improbable that any light can be refracted to the center of the shadow by air at an altitude much over three miles, but the precise altitude will vary according to the dimensions of the shadow-section. Mr. Proctor, in his *Old and New Astronomy*, p. 506, took 2.5 miles for "the depth of atmosphere which is effective in refracting the Sun's light," but

this appears to be a mean value. In the present case the illumination at the center of the shadow could hardly have been greater than that from an annulus of three miles wide, surrounding the Earth, and of intrinsic brilliancy somewhat similar to that of the setting Sun, which, as we have seen, on a particular but fairly typical occasion, had the value 0.00000565 of unabsorbed sunlight. The angular area of this bright ring as seen from the Moon, compared with the angular area of the Sun's disk, must have been:

$$\frac{\left(r+\frac{3r}{3963.3}\right)^2-r^2}{R^2}=\frac{(3240.85)^2-(3238.4)^2}{(952)^2}=\frac{1}{57}$$

Hence the light at the center of the shadow illuminating the eclipsed Moon should have been about $0.00000565\times\frac{1}{57}=0.000000099$ of full moon light if the above assumptions had been fulfilled. The light observed at mid-eclipse was actually about one twenty-fifth of this, confirming the impression that the darkness was greater than usual.

Observation also indicates a notable falling off in the refracted light at about one-half the radius of the shadow, in regard to which it must be remembered that the numbers obtained before and after totality are too large, as the light diffusively reflected from the illuminated segment by the intervening atmosphere, and included in the measurement, forms a considerable, and, for the parts more deeply in shadow, probably the larger proportion of the observed light. The blue gray margin within the shadow near the illuminated segment, no doubt owes its color partly to diffused sky light, although the persistence of this brighter margin for some time after totality has begun, shows that it must be partly due to the greater blueness of the light refracted by the upper air, which has suffered a smaller selective absorption of the shorter waves, and contributes very appreciably to the illumination of the outer part of the shadow. The sudden diminution of light between observations 29 and 33, which were made during totality, is explained by the vanishing of the diffused sky light, at least in part. Theory, however, shows that only the lowest,

densest, and least transparent parts of the Earth's atmosphere could refract the Sun's rays inside the limit of the half radius of the shadow, here 19′. Since the horizontal refraction of 35′ corresponds to an atmospheric pressure of about thirty inches of mercury, the higher layer of air which refracts to the half radius of the shadow must have a pressure of $\frac{19}{35} \times 30 = 8.1$ inches, and air of twice this pressure, corresponding to a height of a little over three miles, will barely refract to the center of the shadow. A diminution of light towards the center of the shadow is therefore a necessary result of the diminishing refraction of the outer layers of our atmosphere, but apparently this change must progress uniformly, and no reason has been suggested for any sudden variation in its rate.

The only alterations in the apparatus which experience has suggested, are the addition of several of the darkest glasses to increase the range, and the substitution of a lighter intermediate neutral tinted glass, transmitting about 40 per cent., in place of the variety called "*In*" which transmits 20.5 per cent. The photometric method which has been tested on this occasion has been found convenient, and abundantly accurate enough for the measurement of light in a lunar eclipse, where, in addition to sky changes, the necessity of a very great range of luminosity has to be met.

Dark eclipses are by no means infrequent. That of June 10, 1816, is said to have been invisible even with the telescope. That of October 4, 1884, is still fresh in memory. Dr. Copeland, who observed this eclipse with a fifteen-inch telescope, found that occultations of 11th magnitude stars could be readily followed, and all who witnessed the event will remember that the Moon was barely visible to the naked eye, resembling a nebula rather than a sharp-edged disk. Great interest attaches to this eclipse, as it occurred while the earth was still enveloped in the dust-cloud of the Krakatoa eruption, to which the exceptional darkness and absence of the usual red color have been plausibly attributed. Many lunar eclipses, however, must have their light quantitatively determined without trusting to eye estimates and

the recollections of years, before the causes and limits of these variations can be definitely settled. Knowledge is cumulative, and the illustration just given holds out a possibility of the discovery of remote meteoric causes from a critical study of the light of the eclipsed Moon. At the least, it would be desirable that future recurrences of exceptionally dark or bright eclipses should find some one ready to measure the exact amount of the obscuration.

ALLEGHENY OBSERVATORY,
September 20, 1895.

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PRELIMINARY TABLE OF SOLAR SPECTRUM WAVE-LENGTHS. IX.

By HENRY A. ROWLAND.

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5429.349		00	5438.507		000
5429.495		0000	5438.674		0000
5429.637		000	5438.917		0000
5429.717		1	5439.256		0000
5429.911	Fe	6 d?	5439.506		0000
5430.062		0	5439.676		0000
5430.295		0000 N	5439.914	A	000
5430.572		00	5440.130		000
5430.993		0000	5440.707		0000
5431.266		0000	5440.861		000
5431.590		0000	5441.031		0000
5431.747	A?	000	5441.188		000
5432.061		0000	5441.350		000
5432.564		000	5441.549	Fe?	1
5432.753	Mn[1]	1 N d?	5442.155		0000
5432.951		0000	5442.499	A?	0000
5433.160	Fe	2	5442.628	Cr	00
5433.406		000 N d?	5442.971		0000 N
5433.614		00 N	5443.179		000
5433.844		00 N	5443.405		0000
5434.384		0000 N	5443.629		000
5434.740 s	Fe	5	5443.825		000
5435.069		0000	5444.108		0000
5435.246		000	5444.300		0000
5435.388		00	5444.796	Co	00
5435.790		0000	5444.933		0000
5435.906		0000	5445.080		0000
5436.071	Ni	2	5445.259	Fe	4
5436.265		0000	5445.621		0000
5436.341		0000	5445.710		0000
5436.508	Fe	1	5446.436		0000
5436.650		0000	5446.577		0000
5436.802	Fe	1	5446.797	Ti	2
5436.938		000	5447.130 s	Fe	6 d?
5437.300	Ni?	000	5447.454		0000
5437.413		00	5447.737		000
5437.591		0000	5447.889		0000
5437.766		0000	5448.142		0000
5437.996		0000	5448.304	A?	0000
5438.259		000	5448.582		00

[1] Compound lines in both Mn and Sun.

[2] A means "atmospheric line."

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5448.882		0000	5462.988		0000
5449.138		000	5463.174 s	Fe	3
5449.369		000	5463.318		0000
5449.913		0000	5463.494 s	Fe	3
5451.005	Sr	00	5463.686		0000
5451.132		0000	5463.841		0000
5451.330		000	5464.037		0000
5452.160		0000	5464.179	Cr	000
5452.309		00	5464.314		0000 N
5452.504		000	5464.490	Fe	0
5452.817		0000	5464.678		0000 N
5453.056		00 N	5465.351	A?	000 N
5453.293		0000	5465.584		0000
5453.444		00	5466.237		0000 N
5453.860		000	5466.409		0000
5454.064		000	5466.609 s	Fe	3
5454.199		00	5466.793		0000
5454.326		000	5466.970		0000
5454.569		0000	5467.198	Fe	1
5454.783	Co	00	5467.355		0000
5454.998		0000	5467.482		0000
5455.297		0000	5467.605		000
5455.671 s	Fe?	2	5467.773		0000
5455.834 s	Fe	4	5467.989		000
5456.117		0000 N	5468.149		0000
5456.319		0000 N	5468.320		00
5456.571	A?	000	5468.601		000 Nd?
5456.734		000 N	5468.843		000
5457.088		0000	5469.000		0000
5457.311		000	5469.278		0000
5457.453		0000	5469.485		00 N
5457.640	Mn	000	5469.972		0000
5457.701		000	5470.298		00
5458.034		0000 N	5470.432	A?	0000
5459.072		0000	5470.650		000
5459.406	A?	000	5470.802	Mn	0
5459.593		0000	5470.883		0
5460.266		0000 N	5471.055		0000
5460.572		000	5471.414	Ti	000
5460.721		00	5471.601		0000
5460.904		0000 N d?	5472.128		0000 N
5461.088		00	5472.503		000
5461.349		0000 N	5472.685		0000Nd?
5461.602		0000 N	5472.916	Fe	1
5461.762		0	5473.135		0000
5462.027		000 N	5473.211		0000
5462.269	A?	0000 N	5473.373		00
5462.478		0000	5473.594		000
5462.705 s	Ni	1	5473.758		000 N
5462.866		0000	5473.950		0000

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5474.113	Fe	3	5484.517		0000
5474.299		0000	5484.846		000
5474.436	Ti?	00	5485.023		0000
5474.668		000	5485.266		0000
5474.962		0000 N	5485.580		0000
5475.286		0000 N	5485.757		000
5475.645		000 N	5485.915		0000
5475.938		0000 N	5486.016		000
5476.217		0000 N	5486.320		000
5476.387		00	5486.730		000
5476.500	Fe	1	5486.968		000
5476.778	Fe	3	5487.170		000 N
5476.946		0000	5487.354	Fe	1
5477.123 s	Ni	5	5487.534	V	0000 N
5477.290		0000	5487.724	Ti	00 N
5477.486		0000	5487.959 s	Fe	3
5477.707		000 N d	5488.139		000
5477.901	Ti	00	5488.374		00 Nd?
5477.992		000	5488.547		0000
5478.160		0000	5488.717		0000
5478.360	A?	0000	5489.194		00
5478.578		00	5489.893		000
5478.668	Fe	0	5490.074		00
5478.900		0000	5490.207		0000
5478.991		000	5490.367	Ti	0
5479.234		0000	5490.531		0000
5479.451		0000 N	5490.672		0000
5479.983		0000	5490.905		0
5480.185		000	5491.037		000
5480.408		0000	5491.897		0000
5480.562		0000	5492.049		00
5480.722	Cr	00 N	5492.242		0000
5480.964		000	5492.424		000
5481.071	Fe	1	5492.566		000
5481.273		000 N	5493.095		00
5481.452	Fe	1	5493.274		00c0
5481.652	Fe, Ti	1	5493.450		00
5481.815		0000	5493.558		0000
5481.941		0000	5493.709	Fe	1
5482.078		00	5493.865		0000
5482.200		0000	5494.063		0 d
5482.469		000 N	5494.193		0000
5482.807		000 N	5494.358		0000
5483.131		0000	5494.536		0000
5483.307	Fe	1	5494.679	Fe	0
5483.566	Co	1 d?	5494.912		000
5483.758		0000	5495.096	Ni	00
5483.884		000	5495.916		000
5484.112		000	5496.119		0000
5484.244		000	5496.458		00c0

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5496.780		00	5507.196		0000
5497.011		000	5507.985		000
5497.192		0000	5508.289		0000 d?
5497.325		000	5508.451		0000
5497.447		0000	5508.625		0
5497.624		0000	5508.840		0
5497.735 s	Fe	5	5509.058		0000
5497.911		0000	5509.321		0000
5498.105		000	5509.756		00
5498.394		0000	5509.938		000 N
5498.566		0000	5510.120	V	0
5498.955		0000	5510.229	Ni	1
5499.234		000	5510.449		0000
5499.376		0000	5510.586		0000
5499.635		000	5510.829		00
5499.804		00	5510.942		00
5499.976		0000	5511.173		0000
5500.560		0000	5511.375		0000
5500.811		0000	5511.644		000
5500.957		0000	5511.872	Fe	0000
5501.193		000	5512.013		00
5501.461	La	0000 N d?	5512.266		00
5501.683 s	Fe	5	5512.470	Fe	1
5501.919		0000	5512.620		00
5502.088		0000	5512.741	Ti	2
5502.297		00	5512.923		0000
5502.480		0000	5513.025		0000
5502.783		0000	5513.198 s	Ca	4
5502.955		0000	5513.439		0000
5503.153		00	5513.592		0000
5503.286	Fe	1	5513.769		0000
5503.444		00	5513.926		0000
5503.710		00 N	5514.057		0000
5503.927		000 N	5514.189		0000
5504.117	Ti	0	5514.431		0000
5504.310		00	5514.563	Ti	2
5504.431		0000	5514.753	Ti	2
5504.599		00	5514.902		0000
5504.867		0000	5515.006		000
5505.099		0000	5515.150		0000
5505.489		0000	5515.317		0000
5505.747		0000 N	5515.565		000
5505.939		0000	5515.697		0000
5506.095	Mn	1	5515.857		00
5506.243		0000	5516.051		0000
5506.395		000	5516.253		0000 N
5506.576		000	5516.513		000
5506.719		000	5516.708		000
5506.823		0000	5516.950	Mn	0
5507.000 s	Fe	5	5517.034	Mn	0

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5517.286	Fe	0	5527.796		000 d
5517.458		0000	5528.084		0000 N
5517.589		0000	5528.297		000
5517.756		00	5528.641 s	Mg	8
5517.984		0000 N	5529.115		00
5518.307		000	5529.384	Fe	00
5518.378		0000	5529.564		000
5518.576		0000	5530.001		0000
5518.754		0000	5530.178		000 d
5519.001		0000	5530.491		0000
5519.289		0000	5530.704		00 N
5519.633		000	5530.997	Ti	00 N
5519.797	Fe	0	5531.321		0000
5520.069		000 N	5531.913		0000
5520.246		0000	5532.202		00
5520.434		000 N	5532.353		000
5520.727		00 N	5532.571		000 N
5520.920		0000	5532.968		1
5521.160		00 N	5533.092		0
5521.349		00	5533.248		000
5521.511		00	5533.365		0000
5521.646		000	5533.654		000
5521.799		00	5533.791		000
5522.002		0000 N	5534.011		000 d
5522.407		0000 N	5534.504		000
5522.665	Fe	2	5534.620		0000
5522.883		0000 N	5534.895	Fe	000
5523.176		0000 N	5535.061 s	Fe	2
5523.469	Ti	000	5535.277		0000 N
5523.550	Ti	000	5535.399		000 N
5523.783		000 N	5535.644	Fe	2
5523.960		0000	5535.778		0
5524.077		0000	5535.985		0000
5524.209		00	5536.072		0000
5524.482		000	5536.300		000
5524.684		000	5536.492		000
5524.785		0000	5536.679		0000
5525.010		000	5536.811		00
5525.220		000	5537.031		0000
5525.347		00	5537.139		0000
5525.565		0000 N	5537.332		000
5525.765	Fe	2	5537.509		0000
5525.929		000	5537.726		0000 Nd?
5526.066		000 N	5537.928	Mn	00
5526.405		000 N	5538.025	Mn	00
5526.789		000 N	5538.253		0000
5527.033		3	5538.405		0000
5527.208		000	5538.526		0000
5527.325		0000	5538.738	Fe	1
5527.619		0000	5538.925		0000

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5539.273		0000	5551.995		0000 N
5539.507	Fe	0	5552.193	Mn	000
5539.745		0000 N d?	5552.452		000
5540.044		000	5552.673		0000Nd?
5540.192		0000	5552.916		00
5540.393		000 N	5553.068		0000
5540.661		000 N	5553.346		000 N
5540.940		000	5553.452		000 N
5541.110		000	5553.619		0000 N
5541.501		0000	5553.804	Fe	1
5541.805		0000	5553.927	Ni	0
5542.361		0000	5554.161		0000
5542.545		0000	5554.459		0000 N
5542.756		0000	5554.748		0000
5542.969		0000	5554.879		0000
5543.112		0000	5555.036		00
5543.262		000	5555.122 s	Fe	3
5543.414 s	Fe	2	5555.395		0000
5543.626		0000	5555.572		0000
5543.975		0000	5555.683		0000
5544.157 s	Fe	2	5555.860		000
5544.386		000	5555.951		0000
5544.560		0000	5556.418		0000 N
5544.831	Y	000	5556.691		0000
5544.983		0000	5556.933		0000 N
5545.266		0000	5557.196		000
5545.487		0000	5557.287		000
5545.643		0000	5557.710		000
5545.913		0000 N d?	5557.946		000 N
5546.147		000 } d	5558.136		0
5546.248		000 }	5558.209	Fe	0
5546.562		0000	5558.391		0000
5546.732	Fe	2	5558.472		0000
5546.956		000	5558.816		0000
5547.215	Fe, V	1	5559.068		000 Nd?
5547.522		0000 N d?	5559.281		000 Nd?
5547.910		0000 N	5559.867		00
5548.161		0000	5560.106		000
5548.413		0000	5560.243		0000
5548.535		0000	5560.434	Fe	2
5548.697		0000	5560.647		0000
5549.546		0000	5560.911		0000
5549.748		0000	5561.234		0000
5549.875		00	5561.464		00
5550.179		00	5561.699		0000 N
5550.873		0000	5561.821		0000 N
5551.110		0000	5562.044		0000 N
5551.241		0000	5562.343		000
5551.524		0000 N	5562.500		0000
5551.767		000 N	5562.717		0000

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5562.933	Fe	2	5575.310		0000
5563.144		0000 N	5575.614		0000
5563.506		000 N	5575.766		0000
5563.623		0000	5575.897		0000
5563.824	Fe	3	5576.079		0000
5563.916		0	5576.320 s	Fe	4
5564.352		0000 N	5576.589		0000
5565.191		0000	5577.252		00
5565.700	Ti	00	5577.561		000
5565.931	Fe	3	5577.783		0000
5566.172		0000 N	5578.739		0000
5566.304		00	5578.946	Ni	1
5566.456		0000	5579.260		0000
5566.629		0000	5579.381		0000
5566.780		0000	5579.574		000
5566.947		000	5579.711		0000
5567.035		000	5580.530		0000
5567.205		0000 N	5580.672		000
5567.367		0000 N	5580.879		0000
5567.501		000	5581.273		0000
5567.621	Fe	2	5581.502		0000
5567.802		0000	5581.740		0000
5567.989		000 Nd?	5581.922		0000 N
5568.297		000	5582.198 s	Ca	4
5568.490		0000	5582.367		0000
5568.682		0000	5582.506		000
5568.925		0000	5582.630		0000
5569.088		00	5582.973		0000
5569.249		0000	5583.186		000
5569.370		000	5583.358		0000
5569.542		0000	5583.607		0000 N
5569.848 s	Fe	6	5583.845		000 N
5570.286		000	5584.208		00 N
5570.615		000	5584.532		000
5570.827		000	5584.729		000
5570.982		0000	5584.988	Fe	0
5571.708		0000	5585.260		0000
5572.370		000 N	5585.397		000
5572.572		0000 }	5585.543		0000
5572.669		0000 }	5585.726		0000
5572.871		000	5585.877		000
5573.075	Fe	6	5586.222		0000
5573.328		1	5586.497		000
5573.524		0000	5586.900		000 N
5573.762		0000	5586.991	Fe	7
5573.876		000	5587.355		0000
5573.974		000	5587.588		0000Nd?
5574.616		000	5587.800	Fe	0
5574.834		0000	5587.947		0000
5575.128		000	5588.084	Ni	1

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5588.368		000	5602.995 } s	Fe	1
5588.469		0000	5603.083 }	Ca	3
5588.985 s	Ca	6	5603.186 }	Fe	4
5589.227		000	5603.516		0000
5589.429		0000 N	5603.738		0000
5589.582	Ni	0	5603.993		00
5586.794		0000 Nd?	5604.416		0000
5590.079		00	5605.171		000
5590.343 s	Ca	3	5605.560		0000
5590.589		000	5605.864		000
5590.725		0000	5606.122		000
5590.927	Ti	000	5606.268		0000
5591.039	Ti	000	5607.220		000
5591.226		0000	5607.372		0000
5591.586		000	5607.614		000
5592.192		0000	5607.761		0000
5592.375	Ni	0	5607.887		00
5592.487	Fe, Ni	1	5608.059		0000
5592.643		000	5608.393		000
5592.881		000	5608.525		000
5593.458		0000	5608.930		0000 N
5593.680		000	5609.196		00
5593.961	Ni	0	5609.395		0000
5594.196		0000 N	5609.901		0000
5594.383		0000 N	5610.023		000
5594.691 s	Ca	4	5610.205		000
5594.884	Fe	1	5610.467		000
5595.112		0000 N	5610.609		0000
5595.284		000	5611.584		00
5595.704		0000	5611.855		000
5595.906		0000	5612.573		000
5596.129		0000	5612.710		000
5596.402		000	5613.929		000 N
5596.555		0000	5614.253		0000 N
5597.290		000	5614.497		00 N
5597.465		0000	5614.632		0000
5598.524 s	Fe	1	5614.819		0000
5598.711 s	Ca	4	5614.997	Ni	0
5599.034		000	5615.199		0000
5599.170		0000	5615.382		0000
5600.243		00 } d	5615.520 s	Fe	2
5600.318		0000 }	5615.751		0
5600.450	Fe	0	5615.877 s	Fe	6
5600.678		0000	5616.404		000
5601.037		0000	5616.541		0000
5601.505 s	Ca	3	5617.163		0000
5601.654		0000	5617.365		00
5602.042		00 Nd?	5617.451		00
5602.296		000	5617.633		0000
5602.783		000	5617.765		0000

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5617.972		0000	5630.091		0000
5618.130		000	5630.313		000
5618.292		0000 N	5630.516		0000
5618.646		000 N	5631.194		0000 N
5618.858	Fe	1	5631.376		0000
5619.060		000 N	5631.720		0000
5619.192		0000 N	5631.913		000 N
5619.455		000	5632.049		000 N
5619.637		0000	5632.232		000
5619.824		0	5632.672		000 Nd?
5620.037		000	5632.971		0000
5620.250		000	5633.441		000 Nd?
5620.452		0000 Nd?	5633.659		0000 N
5620.634		000	5633.866		0000
5620.715	Fe	0	5633.967		0000
5620.862		0000	5634.171 s	Fe	3
5621.438		000 N	5634.446		0000 Nd?
5621.595		000 N	5634.740		0000
5621.833		000	5635.412	C	0000
5622.450		000	5635.559	C	0000
5622.996		000	5635.736	C	0000
5623.179		00	5636.045	Fe	1
5623.857		0000	5636.218		0000
5624.245 s	Fe	1	5636.339		0000
5624.408		0000 N	5636.455		000
5624.575		0000 N	5636.688		000 N
5624.769 s	Fe, V	4	5636.925	Fe	0
5625.096		000	5637.113		0000 Nd?
5625.304		0000 Nd?	5637.339	F?, Ni	1
5625.541	Ni	0	5637.632	Fe	1
5625.755		000	5637.928		000 N
5625.904		00	5638.488	Fe	3
5625.245		000 N	5638.707		0000
5626.468		0000	5638.980		00
5626.812		0000	5639.575		000
5627.034		000	5639.775		0000
5627.313		000	5640.215		0000
5627.475		0000	5640.397		0000
5627.592		0000	5640.538		0
5627.723		000	5640.721		0000
5627.859	V	00	5640.903		0000
5628.087		0000	5641.206		1
5628.239		000 N	5641.353		000
5628.411		0000	5641.510		0000
5628.571	Ni	00	5641.667 s	Fe	2
5628.867	Cr	00	5641.954		0000 d
5629.099		0000	5642.112	Ni	0
5629.261		000	5642.396		0000
5629.453		0000	5642.605		00
5629.924		0000	5642.846		000

Wave-length	Substance	Intensity and Character	Wave-length	Substance	Intensity and Character
5642.978		00	5657.471		0000
5643.302	Ni	0	5657.667		000
5643.503		0000 N	5657.890		0000
5643.817		0000 N	5658.097 s	Y,–	2
5644.159		000	5658.378		0000
5644.257		00	5658.561		0
5644.365	Ti	0	5658.753	Fe	1
5644.566		000	5658.884	Cr	0
5645.254		000 N	5659.052	Fe	4
5645.830 s	Si	1	5659.326		0000
5646.039		00	5659.545		0000
5646.322		000 N	5659.817		0
5646.530		0000 N	5660.001		0000
5646.904		00	5660.133		0000
5647.461	Ti	00	5660.365		0000
5647.664		0000	5660.527		0000
5647.765		0000	5660.739		00
5647.997		0000	5660.892		0
5648.119		0000	5661.022		0
5648.503		000	5661.240		000
5648.796	Ti	00	5661.418		0000
5648.969		0000	5661.570	Fe?	0
5649.131		0000	5661.712		0000
5649.304		00	5661.833		0000
5649.611	Cr	00 N	5662.025		0000
5649.898	Fe-Ni	0 d	5662.200		0000
5650.209	Fe	1	5662.374	Ti	0
5650.420		0000	5662.526		0000
5650.501		0000	5662.744 s	Fe	4
5650.669		0000	5662.967		0000 N
5650.911	Fe	1	5663.155	Ti, Fe, Y	1
5651.099		0000	5663.341		0000
5651.256		0000	5663.735		0000
5651.489		0000	5664.039		0000
5651.691	Fe	0	5664.218	Ni-Cr	1 N
5651.954		0000 N d?	5664.413		0000
5652.239		0000	5664.575		0000
5652.542	Fe	1	5664.797		000
5653.387		0000	5665.557		0000
5653.898		0000	5665.775	Si	1 N
5654.091	Fe	1	5666.131	A?	000
5654.231		0000	5666.899		0
5654.716		00	5666.998		000
5654.991		0000 N	5667.368		0
5655.156		00 N	5667.542		000
5655.395	Fe	1	5667.739	Fe	2
5655.558		0000	5667.997		0000
5655.715 s	Fe	2	5668.300		0000
5655.908		0000	5668.593	V	000
5657.107		0000	5669.130		0000

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Minor Contributions and Notes.

NOTE ON PASCHEN'S LAWS OF RADIATION.

In his paper, "On the Existence of Law in the Spectra of Solid Bodies" (this Journal, October, 1895), Dr. Paschen has propounded two principles of radiation, the first concerning the form of a spectral energy-curve near the maximum, the second connecting the position of this maximum with the absolute temperature. The mathematical expressions of these laws are beautifully simple, and, if true, would be exceedingly convenient.

According to the first principle, the square of the wave-length of the maximum in a normal spectral energy-curve is equal to the product of the wave-lengths of any two points of equal energy on either side of the maximum, provided the ratio of the extreme wave-lengths be not greater than about 1 : 2.

This proposition seems to be approximately true for the spectral energy-curve of a solid body at low temperatures where the maximum has a considerable wave-length, but at high temperatures the normal curve, plotted on the wave-length scale, becomes quite unsymmetrical, and the relation is no longer true. In any case, the determination of the position of the maximum is not susceptible of very great accuracy, since slight errors of observation of the energies near the maximum may displace its position to an extent which, at low temperatures, is comparable with the deviation from the proposed law.

I have tested this law by applying it to a series of measures of the radiation from the crater of the positive carbon of a very powerful arc-light whose infra-red spectrum was observed at the Allegheny Observatory (see the *Am. Jour. Sci.*, **38**, 438, December, 1889). Reducing the measures to the normal scale, the following positions are selected from the smoothing curve of interpolation :

Observed maximum		Energy = 1.000	$\lambda_{max.}$		$= 1^{\mu}.16$
$\lambda_1 = 1^{\mu}.00$,	$\lambda_2 = 1^{\mu}.65$	" = .821	"	(computed)	1 .28
$\lambda_1 = 0$ .90,	$\lambda_2 = 2$ .54	" = .503	"	"	1 .51
$\lambda_1 = 0$ .80,	$\lambda_2 = 3$ .51	" = .281	"	"	1 .68

Here the progressive departure of the computed from the observed

position of the maximum energy, as the limits widen, is too great to be neglected. Even larger departures may be obtained from observed solar curves, but these having been altered by atmospheric absorption are not appropriate for demonstration.

Dr. Paschen's second proposition is that the wave-length of the maximum in the normal spectral energy-curve is inversely proportional to the absolute temperature of the radiating body, or that the product

$$\lambda_{max.} \times T = 2700 \text{ (a constant).}$$

An observed series between the absolute centigrade temperatures 501° and 1282°, showing that the product $\lambda_{max.} \times T$ diminishes at the lower temperatures, is explained by Dr. Paschen as due partly to the fact that the radiating body, iron oxide, is not absolutely black, and radiates less of the longer waves than a black body; the deviation is also partly attributable to the fact that the blackened bolometer does not absorb the long waves as completely as it does the short ones; and in addition to these causes which tend to diminish the assigned wave-length of the maximum at the lower temperatures, it is considered probable that errors in the determination of the shorter wave-lengths, corresponding to the position of the maximum at higher temperatures, have caused the assignment of too large values to the wave-lengths of these maxima.

Nevertheless, in view of the known complexity in the radiation of a solid body, and the various rates of increment with the temperature attaching to different rays, it is improbable that the law connecting the position of the maximum and the temperature should be as simple as this, which predicates constancy in the product $\lambda_{max.} \times T$; and the following determinations, through a wider range of temperature than that employed by Paschen, show that the value of the product $\lambda_{max.} \times T$, for the almost absolutely black substance carbon, increases with the temperature.

Messrs. Wilson and Gray (*Proc. R. S.*, **58**, 35, July, 1895) have determined the temperature of the hottest part of the positive pole of the electric arc as 3600° absolute. The wave-length of the maximum radiation from the same is about $1^{\mu}.16$, as already stated. The product, 4176, is much larger than Paschen's constant.

For the lowest temperature I choose an observation made at the Allegheny Observatory in very cold and dry winter weather, requiring no correction for the atmospheric absorption which is so strongly

exerted at the position of the maximum for bodies at low temperatures. I estimate the wave-length of the maximum, using the values which Paschen has given for a rock-salt prism in the transformation.

For the middle temperature one of Paschen's determinations is selected, taking his highest temperature at which iron oxide behaves more nearly like a black body. We have.

	$\lambda_{max.}$	T	$\lambda_{max.} \times T$
Carbon of electric arc -	1$^{\mu}$.16	3600.°	4176.
Iron oxide - - -	2 .125	1282.	2727.
Lampblack - - -	7 .3	318.	2321.

If these products are plotted with the wave-lengths as abscissæ, it will be seen that the value of $\lambda_{max.} \times T$ can hardly be less than 10,000 for a wave-length of 0$^{\mu}$.5 (corresponding to the maximum in the normal solar spectrum), which would give 20,000.° as the minimum value of the solar temperature.

As will be recognized by all who are familiar with attempts to reach a value far above the limit of actual observation, there is great uncertainty as to whether a law, which apparently holds at lower temperatures, will continue to be followed at higher ones, and the estimate just given is open to the same objection which affects all such work, that it transcends actual knowledge. FRANK W. VERY.

PHOTOGRAPHIC MAPS OF METALLIC SPECTRA.

WE are glad to call attention to the following statement by Professor Henry Crew in regard to photographic maps of metallic spectra which he has prepared for distribution. We have carefully examined the maps, and can testify to their excellence.

NORMAL SPECTRUM OF THE MAGNESIUM ARC.

A map of the arc spectrum of Magnesium has recently been completed for the region lying between λ 2600 and λ 5800. The photographs have been made by use of a Rowland concave grating having a radius of ten feet. All the plates, except one, lie in the second order spectrum. The scale is approximately three Ångström units to the millimeter.

The metallic arc is formed between moving poles after the manner devised by Messrs. Crew and Tatnall, and described in the *Philosophical Magazine*, October, 1894. In this way, the continuous spectrum of the heated poles and the carbon of the customary arc are avoided.

On the upper side of the Magnesium spectrum is photographed a scale of Ångström units; on the lower side, and in immediate juxtaposition, is photographed the spectrum of the Iron arc. On the back of each plate is printed a list of Kayser and Runge's values for the wave-lengths included between the extremities of that plate; so that almost any line in their list can be identified at once. It being impossible to show all the strong lines and all the weak lines on one plate, the preference has been shown to the weaker lines, *i. e.*, the stronger lines are often over-exposed.

Only commercial Magnesium has been used in the preparation of these photographs. As a consequence many impurities make their appearance. But it is believed that this feature will add to the usefulness of the map: for most of these impurities have been identified and are printed, with their wave-lengths, on the back of the mount. Naturally the impurity lines are very weak, and many that appear on the negative cannot be seen on the print.

The prints are on "Lithium paper," and measure eleven inches in length, each thus covering eight hundred Ångström units. Each portion of the spectrum appears on two different plates, since the wave-length of the middle of one plate differs from the wave-length of the middle of the next plate by only four hundred Angström units.

The list of the plates is as follows:

No. of Plate	Region	Remarks
1.	λ 5100. to λ 5900.	Showing λ 5711.374, the longest wave-length given by Kayser and Runge.
2.	λ 4600. to λ 5400.	Showing reversal in Fraunhofer's "b" group, the first triplet in the "*Zweite Nebenserie*" of Kayser and Runge.
3.	λ 4100. to λ 4900.	Showing the line at 4481. which Scheiner suggests may be a criterion of stellar temperature.
4.	λ 3600. to λ 4400.	Showing the only triplet of the "*Erste Nebenserie*" which lies in the visible spectrum.
5.	λ 3200. to λ 3900.	Showing one triplet from each series, thus illustrating the difference in physical character between the two.
6.	λ 2770. to λ 3400.	Showing five triplets and the remarkable reversal of λ 2852.
7.	λ 2400. to λ 3800.	A first order plate intended only for the region between λ 2600, and λ 2800; very unsatisfactory.
8.	λ 4600. to λ 5400.	Showing the Mg band at λ 5007. as seen in the arc.

These silver prints are each mounted on white cardboard, 2½ × 12

inches. This size will be found a very convenient one for use at the eye-end of the spectroscope and for purposes of instruction in general.

The set of eight photographs complete will be sent to any address, postpaid, for three dollars.

All orders should be sent to Business Agent, Northwestern University, Evanston, Ill., U. S. A.

NOTICE.

It is proposed to complete, at a date as early as possible, maps of all the other elements that can be worked by this method.

Maps of *Zinc* and *Aluminum* will be finished and ready for distribution within a few months.

November, 1895.

HARVARD COLLEGE OBSERVATORY, CIRCULAR NO. 1.

INTRODUCTION.

For some years the need has been felt at the Harvard College Observatory of some means of making a more prompt announcement of the results of its work. It is proposed therefore to issue a series of circulars, as required, to announce any matters of interest, such as discoveries made here, the results of recent observations, new plans of work, and gifts or bequests. It is not proposed to give these circulars a wide distribution, but rather to use them as a means of bringing new facts to the attention of the editors of astronomical and other periodicals, and thus secure the immediate publication of such portions as would be of interest to the readers of these periodicals. The distribution will be made without charge to such persons as will be likely to use the results. Editors who have published extracts from the circulars and desire their continuance are requested to signify this by sending to this Observatory marked copies of the publications in which the extracts appear.

A NEW STAR IN CARINA.

From an examination of the Draper Memorial photographs taken at the Arequipa Station of the Observatory, Mrs. Fleming has discovered that a new star appeared in the constellation Carina in the spring of 1895. A photograph, B 13027, taken on April 14, 1895, with an exposure of 60 minutes, shows a peculiar spectrum in which the hydrogen lines $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, are bright, and the last four of

these are accompanied by dark lines of slightly shorter wave-length. A conspicuous dark line also appears about midway between H_γ and H_δ. A comparison of the spectrum of this star with that of Nova Aurigae and Nova Normae shows that all three closely resemble each other and are apparently identical in their essential features. Another photograph taken on June 15 with an exposure of 60 minutes shows a change in the spectrum of this object. The hydrogen lines H_β, H_γ, and H_δ, are still bright although the continuous spectrum is very faint. Another line whose wave-length is about 4700 is here as bright as the hydrogen lines. On the photograph taken on April 14 it is barely visible.

An examination was next made of all the photographs of the region containing this star. On sixty-two plates, the first taken on May 17, 1889, and the last on March 5, 1895, no trace of the star is visible, although on some of them stars as faint as the fourteenth magnitude are clearly seen. The exposures of these plates varied from 10 to 242 minutes. On nine plates, the first taken on April 8 and the last on July 1, 1895, the star appears and its photographic brightness diminishes during that time from the eighth to the eleventh magnitude. This star precedes *A. G. C.* 15269 (photometric magnitude 5.47) $0^m.5$, and is $0'.7$ north. Its approximate position for 1900 is therefore in R. A. $11^h\ 3^m.9$; Dec.—61° 24′. Two stars of the eleventh magnitude are near the Nova. One is nearly north, 110″ distant, the other is 80″ south preceding.

EDWARD C. PICKERING.

October 30, 1896.

HARVARD COLLEGE OBSERVATORY, CIRCULAR NO. 2.

VARIABLE STAR CLUSTERS.

PROFESSOR SOLON I. BAILEY, in charge of the station at Arequipa, maintained by this Observatory, has discovered from an examination of the photographs obtained by him of certain globular clusters that they contain an extraordinary number of variable stars. This is not a general condition of stellar clusters, however, for in others similarly examined by Professor Bailey no variable stars have been found. The photographs used in this discussion were taken at Arequipa with the 13-inch Boyden Telescope. In the cluster in Canes Venatici, Messier 3 (*N. G. C.* 5272), no less than eighty-seven stars have been proved to be variable from an examination of fifteen photographic plates. The

change in every case is certain, and has been confirmed independently by Mrs. Fleming and the writer from an examination of six of these plates. Sometimes the variation amounts to two magnitudes or more, and sometimes it does not exceed half a magnitude on the plates which were used for its confirmation. No star was included in this count if either of the three observers doubted the variation. Nine other stars were found to be variable by Mr. Bailey, but they are not included since they did not show sufficient change on the plates used in confirmation. In like manner, from an examination of seventeen plates, Mr. Bailey found forty-six variables in the cluster Messier 5 (*N. G. C.* 5904) which were confirmed on five plates. Fourteen other stars in this cluster are also probably variable but have not yet been confirmed. This cluster is frequently described as 5 M Librae, probably following Smyth. It is actually in Serpens, and very near 5 Serpentis. Two variable stars have been confirmed in *N. G. C.* 7089 from an examination of six plates, three in *N. G. C.* 7099 from five plates, five with small range in *N. G. C.* 362 from three plates, and four in *N. G. C.* 6656 from three plates. On the other hand, a similar examination of two plates of each of the clusters *N. G. C.* 6218, 6397, 6626, 6705 and 6752 failed to detect a single variable star, several hundred stars in each case apparently having exactly the same brightness on both plates. As, however, these plates were taken within a few days of each other, only variable stars of short period could have been detected on them. In general, no variables have been found within about one minute of the center of the clusters on account of the closeness of the stars. None of these variables are more than ten minutes distant from the centers of the clusters. In *N. G. C.* 5904 a circle 110″ in diameter contains sixteen stars, six of which, or nearly 40 per cent., are variable. In the entire cluster about 750 stars were examined and 46 found to be variable, as above stated, so that they form about 6 per cent. of the whole. Of all the stars visible to the naked eye less than 1 per cent. are variable.

In 1890 Mr. Packer discovered two variable stars in the cluster *N. G. C.* 5904 (*English Mechanic*, **51**, 378, *Sidereal Messenger*, **9**, 380, 381; **10**, 107). One of these variables was discovered independently by Mr. Bailey but is not included in the above lists. Several stars in this cluster were thought to be variable by Mr. Common (*Monthly Notices*, **50**, 517; **51**, 226). One of them is too near the center, the others too distant to be included in the above discussion. The varia-

ble star discovered in the cluster *N. G. C.* 5272 by the writer in 1889 is also too near the center to be included.

Some of these variable stars have short periods, not more than a few hours. For instance, one of them, No. 12, which precedes the center of *N. G. C.* 5904 by about three minutes of arc. Five photographs of this cluster were taken on July 1, 1895, at intervals of an hour. The corresponding magnitudes of the variable as derived from these plates are 14.3, 13.5, 13.8, 13.9 and 14.3. Four plates taken on August 9, 1895, also at intervals of an hour, gave the magnitudes 14.2, 14.6, 14.8 and 15.0

Right ascensions and declinations cannot conveniently be used for indicating the individual stars in close clusters. They can only be found readily from photographic or other charts on which they are marked. Such charts are now being prepared for publication in the Annals of the Observatory. Meanwhile marked photographs will be sent to such astronomers as may wish to study them.[1]

EDWARD C. PICKERING.

November 2, 1895.

[1] I have examined some of the negatives of clusters photographed by Professor Bailey, and have had no difficulty in confirming the variability of several stars.

G. E. H.

Reviews.

Zur Theorie der Verbreiterung der Spectrallinien. Fürst B. Galitzin, *Wied. Ann.* **56,** 78–99 (1895).

At first sight, it might appear that a very great deal was known concerning an element when all the lines of its various spectra had been mapped and measured with a high degree of accuracy. And, indeed, this is quite true, especially for those elements whose wave-lengths have been connected by such beautifully simple laws as those of Balmer, and Kayser and Runge.

But, if one cares to realize the wide gap which still separates the science of spectroscopy from that of everyday mechanics, he has only to ask himself the dynamical meaning of the different physical features which these lines present, whether as compared with each other under the same conditions, or with themselves under different conditions.

Within the last twenty years, however, numerous attempts have been made to bridge this chasm. And, when we consider the results, the only matter for surprise is, not that so many facts remain unexplained, but that such widely divergent hypotheses as have been offered are capable, each, of predicting so many actual phenomena. This results partially, perhaps, from the fact that all these ideas are ultimately connected and contain some degree of truth, while all are, probably, still very wide of the mark.

Among these happy conjectures, that of Fürst Galitzin is at once the latest and most novel. His views are presented in the paper under review, which appeared some months ago in the *St. Petersburg Academy* and is reprinted in the current number of *Wiedemann's Annalen.*

An interesting sketch of the earlier work in this field forms a preface to the article. The ideas of Lippich, based on Doppler's principle, are quickly disposed of as incapable of explaining asymmetry in the widening of the lines. Wüllner's modification of Zöllner's theory, based on Kirchhoff's law, next comes up for discussion. Here Kayser's[1] views are adopted *in toto.* The only criticism of the molecular theory is that while it explains, in a sort of general way, the general phe-

[1] *Wied. Ann.* **42,** 310 (1891).

nomena, it does not touch the heart of the question, viz., the mechanics of the molecule.

The much neglected theory which Lommel[1] advanced in 1878 (and which, in an improved form, has been recently advocated by Jaumann) is based on three assumptions, viz., (1) that the vibrating parts oscillate in consequence of a force which is itself periodic; (2) that each of these parts is restored to its position of equilibrium by a force which can be expressed as a function of the positive powers of the distance by which the part is displaced from its position of equilibrium; (3) that the motion of the vibrating part meets with a resistance which is at every point proportional to the speed of the part.

The differential equation which expresses such a motion yields the following well-known integral, an expression which makes its appearance in almost every department of physics.

$$\chi = N \epsilon^{-kt} \sin (r t + \psi)$$

where k is the damping coefficient, r the frequency, t the time, N and ψ constants of integration. The highly original step of Lommel occurs just here, when he shows by analysis that this integral is equivalent to an infinite sum ("*continuum*") of simple sine vibrations, not containing any damping term whatever, but including all possible frequencies from $-\infty$ to $+\infty$. In other words, each line becomes a more or less rapidly shaded continuous spectrum extending to both sides of the wave-length whose frequency is r. At this point, Galitzin makes the very pertinent inquiry, Is this resolution of the damped vibration a fact of nature, or a mere mathematical trick?

The theory of Lommel and Jaumann is clearly and fairly presented. But the charge of incompetency, in numerous particulars, is urged against it with equal clearness and fairness. For instance, the theory of the damped atom offers no explanation of effects due to temperature variations. It explains the asymmetric shading and constant difference in frequency between adjacent members in certain series; but the effects of pressure are accounted for only by a most unlikely hypothesis and one quite unjustified by the kinetic theory of gases. Galitzin goes so far as to say that in the case of thermal equilibrium (say in a Geisler tube where, at each instant, as much energy is supplied to it as is radiated from it), even if the vibration *were* damped, one could not detect it, for the *mean* energy of each radiating atom would be con-

[1] *Pogg. Ann.*, **3**, 251–283 (1878).

stant. Considered as an objection to Lommel's theory, however, the point does not appear to be well taken. For the fact that the mean energy of an atom remains constant is merely equivalent to saying that the intensity of the corresponding line does not vary *in time;* while all that Lommel here attempts is the explanation of the variation of the line *with wave-length, i. e.*, on either side of the maximum.

Still another difficulty advanced against the damped vibration is that the atom involved is one of gross matter. But, so far as your reviewer can see, the nature of the substance which vibrates cuts no essential figure in Lommel's discussion. His equations are rather kinematical than dynamical. And one can imagine him, if confronted by this difficulty, replying "Hypotheses non fingo!"

Galitzin's own views have for a basis the electromagnetic theory of light and the following assumptions: (1) Each luminous molecule or atom is an "exciter," a sort of Hertzian vibrator, of perfectly definite period. (2) It has a constant coefficient of self-induction, L, and a constant capacity, C. (3) But its electrical resistance, R, like that of the Amperian molecule, is zero: since, otherwise, this resistance would produce damping, and the energy of the displacement current would sooner or later be transformed into heat of the kind considered in Joule's law. But no heat of this kind is developed *inside* the molecule; at least, the mechanical theory of heat assumes that *this* heat is due to the motion of the molecule, as a whole, in space, and not a form of energy developed inside the molecular conductor.

Such a molecular resonator would have a free period given by the equation

$$T = 2\pi\sqrt{LC}$$

and its spectrum would be a single sharp line. Such a current if left to itself, in its minute, but perfect, conductor would suffer damping, not on account of the resistance of the conductor, but from loss of energy through electromagnetic radiation.

The author next discusses the case in which *two* of these molecular vibrators are in the field at once, and proceeds to compute the effect of one on the other, assuming for this purpose that their coefficient of mutual induction is a constant during a large number of electrical vibrations in the molecule, an assumption which appears to be amply justified by the kinetic theory of gases. When the differential equations describing this state of affairs are integrated, they give this curious result, viz., each molecule has its own free period changed into

a different one depending upon the coefficient of mutual induction, and in addition has superposed upon it a forced vibration of still a different period. That is to say, each molecule now radiates light of two different wave-lengths, these two lines lying one on each side of the original line.

Another important feature of the solution is this, viz., each of these two periods depends upon the coefficient of mutual induction, and this in turn upon the distance between the two molecules. The nearer the molecules, the more the forced periods differ from the free; the farther apart the molecules, the more the forced periods approximate the free. The connection, then, between increase of pressure and widening of the lines is this: Increase of pressure diminishes the mean distance between the molecules; this, in turn, increases the coefficient of mutual induction, M, between the two. And, since the two periods are (in the case where the molecules are of the same gas)

$$T_1 = 2\pi\sqrt{C(L+M)}$$
$$T_2 = 2\pi\sqrt{C(L-M)}$$

this increase of mutual induction carries with it a change in the period of each of the two vibrations; but the change is such as always to make the divergence between them greater. When, now, the two periods are very nearly the same, as is assumed to be the case, this corresponds to a widening of the line. The relative value of the two amplitudes must determine the asymmetry.

The author next proceeds to discuss, but only in a qualitative way, the case of a gas composed of many molecules of the same kind. But the general conclusions are not very different from those arrived at in the case of only two molecules.

Considered as a first approximation in the explanation of the widening of a single definite line in the spectrum—and this is all the author claims for it—the attempt is interesting and suggestive, and the theory, perhaps, as successful as any yet proposed. However, a case of this kind is eminently one in which suspended judgment is the best judgment. H. C.

Recent Publications.

A LIST of the titles of recent publications on astrophysical and allied subjects will be printed in each number of THE ASTROPHYSICAL JOURNAL. In order that these bibliographies may be as complete as possible, authors are requested to send copies of their papers to both Editors.

For convenience of reference, the titles are classified in thirteen sections.

1. THE SUN.

BIGELOW, F. H. The Status of the Solar Magnetic Problem. Sci **2**, 509–513, 1895.

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HALLOCK, WILLIAM. Bolometric Investigations in the Infra-red Spectrum of the Sun. Sci. **2**, 174–178, 1895.

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NOTICE.

The scope of THE ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

It is intended to publish in each number a bibliography of astrophysics in which will be found the titles of recently published astrophysical and spectroscopic papers. In order that this list may be as complete as possible, and that current work in astrophysics may receive appropriate notice in other departments of the JOURNAL, authors are requested to send copies of all papers on these and closely allied subjects to both Editors.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. If a request is sent *with the manuscript* twenty-five reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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PLATE X

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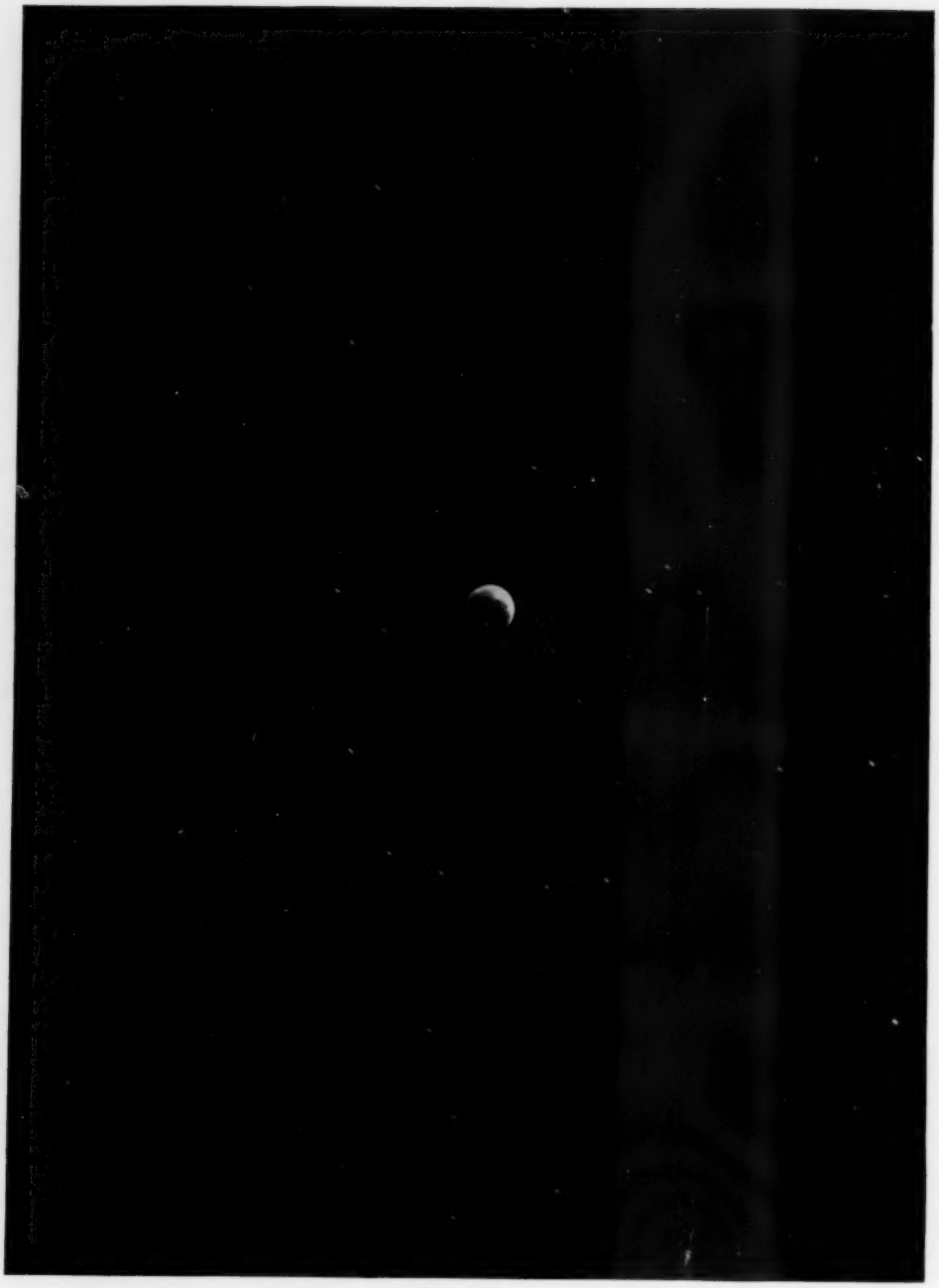

W E

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PHOTOGRAPH OF THE TOTAL PHASE OF THE LUNAR ECLIPSE OF SEPT. 3, 1895

By E. E. BARNARD, Lick Observatory

(9^h 57^m to 10^h 06^m Standard Pacific Time) Six inch Portrait Lens